* cost, time, working hrs, M/c ⇒ Minimiza?

* Profit, Sales, Revenue, η, Run ⇒ Maximiza?

⇒ Assignment = Execution
Xportation = Logistics

Matrix

Matrix is an arrangement of (m×n) numbers into

m rows (Horizontal Lines)
n columns (Vertical Lines)

Square Matrix $A = \begin{bmatrix} 2 & 3 \\ 4 & 8 \end{bmatrix}_{2\times 2}$ $B = \begin{bmatrix} 5 & -1 & 0 \\ 9 & 8 & 7 \\ 3 & 2 & 4 \end{bmatrix}_{3\times 3}$

A is square matrix of order 2
B ——"—— 3

?ngular ?ix $C = \begin{bmatrix} 5 & 9 & 8 \\ -3 & 2 & 1 \end{bmatrix}_{2\times 3}$ $D = \begin{bmatrix} 1 & 9 \\ 2 & 5 \\ 7 & 2 \end{bmatrix}_{3\times 2}$

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Assignment Problem

* Assign jobs to workers / machines

* Assignment is a typical optimization technique practically useful in a simulation where a certain number of tasks are required to be assigned to an equal number of facilities, on a one to one basis, so that the resultant effectiveness is optimize

Probl)

Time in hr

		X	Y	Z
	A	2	4	6
Jobs	B	1	6	7
	C	3	4	5

← workers

Sol^n = The given matrix is a Square Matrix

Step I The rows are reduced

	X	Y	Z	
A	(2-2)	(4-2)	(6-2)	as 2 is mi
B	(1-1)	(6-1)	(7-1)	as 1 is mi
C	(3-3)	(4-3)	(5-3)	as 3 is mi

∴

	X	Y	Z
A	0	2	4
B	0	5	6
C	0	1	2

Step II Columns are reduced

	X	Y	Z
A	(0-0)	(2-1)	(4-2)
B	(0-0)	(5-1)	(6-2)
C	(0-0)	(1-1)	(2-2)
	↓ as 0 is min	↓ as 1 is min	↓ as 2 is min

∴

	X	Y	Z	
A	0	1	2	
B	0	4	4	
C	0	0	0	← 2 zeroes
	← 3 zeroes			

* Draw lines (vertical | Horizontal)
* very first line covering Maximum zeros
* Repeat till all zeroes are covered

→ check :-

(no. of lines drawn) $\overset{?}{\leq}$ (order of matrix)

2 < 3

∴ Solution is not optimum.

te [if (no of lines) = (order of matrix)
then soln is optimum

& (no. of lines) > (order of matrix)
this never happens]

Gr I — numbers not present on any line drawn ⇒ Select the least number from Gr I & <u>SUBTRAC</u> from each member

Gr II — numbers present on only 1 line drawn ⇒ unchanged

Gr III — numbers present on 2 lines drawn ⇒ The least number fro Gr I should be <u>ADDED</u> to each memb

here

Gr I 1, 2, 4, 4
Gr II 0, 0, 0, 0
Gr III = 0

∴ New ~~Matrix~~ values are

	X	Y	Z
A	0	0	1
B	0	3	3
C	1	0	0

∴ Draw vertical / Horizontal lines covering all zeroes.

	X	Y	Z
A	0	0	1
B	0	3	3
C		0	0

Sol^n optimum

here

number of lines drawn = order of matrix

3 = 3

$\therefore$ Solution is optimum

$$
\begin{array}{cccc}
 & X & Y & Z \\
A & \cancel{0} & \boxed{0} & 1 \\
B & \boxed{0} & 3 & 3 \\
C & 1 & \cancel{0} & \boxed{0}
\end{array}
$$

Allocation :- Select a zero which is single zero on any row or column, & Allocate / Assign that. then discard all zeroes on same zero vertically & horizontally

$\therefore$ A → y , B → x , C → z

Minimum time = 4 + 1 + 5 = 10 hrs *

→ Job A is assigned to worker y

Job B ———— " ———— x

Job C ———— " ———— z

* select same place value from problem statement.

Prob. 2

		Workers			
		P	Q	R	S
	A	60	66	63	69
	B	59	67	62	70
Jobs	C	52	60	66	76
	D	56	64	65	63

Sol^n :- The given matrix is a square matrix.

Step I The Rows are reduced

	P	Q	R	S
A	0	6	3	9
B	0	8	3	11
C	0	8	14	24
D	0	8	9	7

Step II The columns are reduced.

	P	Q	R	S
A	0	0	0	2
B	0	2	0	4
C	0	2	1	17
D	0	2	6	0

no of lines drawn = order of matrix = 4

∴ Sol^n is optimum.

Step III

	P	Q	R	S
A	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	2
B	$\cancel{0}$	2	$\boxed{0}$	4
C	$\boxed{0}$	2	1	17
D	$\cancel{0}$	2	6	$\boxed{0}$

∴ A → Q 66
B → R 62
C → P 52
D → S 63

∴ 243 hr.

3

Prob. 4

cost in Rs.

	A	B	C	D
P	78	79	82	76
Q	89	90	95	92
R	80	88	92	95
S	76	84	88	90

Sol^n:- The given matrix is a square matrix.

Step I - The rows r reduced.

	A	B	C	D
P	2	3	6	0
Q	0	1	6	3
R	0	8	12	15
S	0	8	12	14

Step II - The columns are reduced

	A	B	C	D
P	2	2	0	0
Q	0	0	0	5
R	0	7	6	15
S	0	7	6	14

∴ As No of lines drawn < order of matrix

∴ sol^n is not optimum

∴ Gr I Gr III Gr III

Step III

	A	B	C	D
P	8	2	0	0
Q	6	0	0	3
R	0	1	0	9
S	0	1	0	8

∴ 4 = 4

∴ sol^n is optimum

Now assign.

	A	B	C	D
P	8	2	$\cancel{0}$	$\boxed{0}$
Q	6	$\boxed{0}$	$\cancel{0}$	3
R	$\boxed{0}$	1	$\cancel{0}$	9
S	$\cancel{0}$	1	$\boxed{0}$	8

multiple solution.

$\therefore$

$P \rightarrow D$	76	76
$Q \rightarrow B$	90	90
$R \rightarrow A$	80	92
$S \rightarrow C$	88	76
	334	334.

$\therefore$ 334 is the min. cost in Rs.

Time in hr.

Prob 3

	A	B	C	D
P	36	38	47	50
Q	39	48	44	49
R	46	51	47	40
S	48	49	54	56

Sol^n. Rows ⇒

	A	B	C	D
P	0	2	11	14
Q	0	9	5	10
R	6	11	7	0
S	0	1	6	8

Columns ⇒

	A	B	C	D
P	0	1	6	14
Q	0	8	0	10
R	6	10	2	0
S	0	0	1	8

4 = 4 ∴ Sol^n is optimum

Assign ⇒

	A	B	C	D
P	$\boxed{0}$	1	6	14
Q	$\cancel{0}$	8	$\boxed{0}$	10
R	6	10	2	$\boxed{0}$
S	$\cancel{0}$	$\boxed{0}$	1	8

P → A ⇒ 36
Q → C ⇒ 49
R → D ⇒ 44
S → B ⇒ 40

169 Minimum (Time in hr)

Prob 5

	A	B	C	D	E
P	10	20	27	21	30
Q	14	18	20	22	19
R	16	19	26	28	24
S	11	17	25	27	22
T	9	23	22	24	27

Sol^n :- The matrix is square

Rows reduced.

	A	B	C	D	E
P	0	10	17	11	20
Q	0	4	6	8	5
R	0	3	10	12	8
S	0	6	14	16	11
T	0	4	3	5	8

Column Reduced

	A	B	C	D	E
P	0	7	14	6	15
Q	0	1	3	3	0
R	0	0	7	7	3
S	0	3	11	11	6
T	0	1	0	0	3

as 4 < 5 ∴ n't optimum sol^n.

Gr I , Gr II Gr III

$\therefore$

	A	B	C	D	E
P	0	7	11	3	15
Q	0	1	0	0	0
R	0	0	4	4	3
S	0	3	8	8	6
T	3	4	0	0	6

again $4 < 5$

$\therefore$ nt a optimum solⁿ

$\therefore$ Gr I Gr II Gr III

	A	B	C	D	E
P	0	7	8	0	12
Q	3	4	0	0	0
R	0	0	1	1	0
S	0	3	5	5	3
T	6	7	0	0	6

as $5 = 5$ $\therefore$ the optimum solⁿ

$\therefore$

	A	B	C	D	E
P	$\cancel{0}$	7	8	$\boxed{0}$	12
Q	3	4	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$
R	$\cancel{0}$	$\boxed{0}$	1	1	$\cancel{0}$
S	$\boxed{0}$	3	5	5	3
T	6	7	$\boxed{0}$	$\cancel{0}$	6

$\therefore$ $P \to D$, $Q \to E$, $R \to B$, $S \to A$, $T \to C$

$\therefore$ $21 + 19 + 19 + 11 + 22 = 92$

40
30
22

Prob

Time in days
workers

jobs		P	Q	R	S
	A	64	67	63	62
	B	60	68	69	59
	C	59	70	65	58
	D	67	65	60	67

Sol^n = Given matrix is a square matrix.

step I) The rows are reduced.

	P	Q	R	S
A	2	5	1	0
B	1	9	10	0
C	1	12	7	0
D	7	5	0	7

Step II) The columns are reduced.

	P	Q	R	S
A	1	0	1	0
B	0	4	10	0
C	0	7	7	0
D	6	0	0	7

Now

	P	Q	R	S
A	1	0	1	0
B	0	4	10	0
C	0	7	7	0
D	6	0	0	7

No. of lines drawn = order of matrix

$\therefore$ 4 = 4

$\therefore$ optimum solution.

we

* very 1st line must cover maximum no. of zeros

	P	Q	R	S
$\therefore$ A	1	$\boxed{0}$	1	$\cancel{0}$
B	$\boxed{0}$	4	10	$\cancel{0}$
C	$\cancel{0}$	7	7	$\boxed{0}$
D	6	$\cancel{0}$	$\boxed{0}$	7

selecting zero :- select row/column with 1 zero & cancel all other with vertically & horizontally

[job A assigned to worker Q]

$\Rightarrow \therefore$

A $\rightarrow$	Q	67
B $\rightarrow$	P	60
C $\rightarrow$	S	58
D $\rightarrow$	R	60

select same place in problem statement

$\therefore$ Minimum time (days) $= 67 + 60 + 58 + 60$
$= 245$ days.

↳ if no parameters are given then problem is of 'minimization'

Prob)

	P	Q	R	S	T
A	6	9	13	10	8
B	8	11	14	11	8
C	9	13	10	13	11
D	11	15	12	9	14
E	18	17	10	9	13

Sol^n - Given matrix is square matrix

Step I → The rows are reduced

	P	Q	R	S	T
A	0	3	7	4	2
B	0	3	6	3	0
C	0	4	1	4	2
D	2	6	3	0	5
E	9	8	1	0	4

Step II → The columns are reduced.

	P	Q	R	S	T
A	0	0	6	4	2
B	0	0	5	3	0
C	0	1	0	4	2
D	2	3	2	0	5
E	9	5	0	0	4

Now

	P	Q	R	S	T
A	0	0	6	4	2
B	0	0	5	3	0
C	0	1	0	4	2
D	2	3	2	0	5
E	9	5	0	0	4

∴ No. of lines drawn = order of matrix

5 = 5

∴ solution is optimum

∴

	P	Q	R	S	T
A	~~0~~	$\boxed{0}$	6	4	2
B	~~0~~	~~0~~	5	3	$\boxed{0}$
C	$\boxed{0}$	1	~~0~~	4	2
D	2	3	2	$\boxed{0}$	5
E	9	5	$\boxed{0}$	~~0~~	4

∴ A → Q ⇒ 9

B → T ⇒ 8

C → P ⇒ 9

D → S ⇒ 9

E → R ⇒ 10

(+)

∴ Optimum Sol^n 45

Prob)

	A	B	C	D
P	101	105	107	108
Q	99	104	109	109
R	108	100	112	116
S	111	106	113	115

Sol^n → Nothing specified in problem
∴ problem for minimization.

Given matrix is square matrix

Step I

Rows r reduced

	A	B	C	D
P	0	4	6	7
Q	0	5	10	10
R	8	0	12	16
S	5	0	7	9

Step II columns are reduced.

	A	B	C	D
P	0	4	0	0
Q	0	5	4	3
R	8	0	6	9
S	5	0	1	2

Now

	A	B	C	D
P	0	4	0	0
Q	0	5	4	3
R	8	0	6	9
S	5	0	1	2

No. of lines < order of matrix
3 4

∴ sol^n is not optimum.

	A	B	C	D
P	~~0~~ 1	5	0	0
Q	0	5	3	2
R	8	0	5	8
S	5	0	0	1

No. of lines = order of matrix
4 = 4

ssigning :-

	A	B	C	D
P	~~0~~ 1	5	~~0~~	$\boxed{0}$
Q	$\boxed{0}$	5	3	2
R	8	$\boxed{0}$	5	8
	5	~~0~~	$\boxed{0}$	

ing

P → D
Q → A ⟹
R → B
S → C

108
99
100
113

420 is optimum sol^n for minimization.

Minimum Value

✓

Prob)

	P	Q	R	S	T	U
A	80	82	75	68	69	79
B	68	69	81	67	70	75
C	77	65	76	70	68	74
D	72	71	75	77	73	73
E	76	78	71	73	76	70
F	70	74	79	78	77	79

Sol^n $\rightarrow$ The give matrix is a square matrix.

I) Rows

	P	Q	R	S	T	U
A	12	14	7	0	1	11
B	1	2	14	0	3	8
C	12	0	11	5	3	9
D	1	0	4	6	2	2
E	6	8	1	3	6	0
F	0	4	9	8	7	9

II Columns.

	P	Q	R	S	T	U
A	12	14	6	0	0	11
B	1	2	13	0	2	8
C	12	0	10	5	2	9
D	1	0	3	6	1	2
E	6	8	0	3	5	0
F	0	4	8	8	6	9

Lines drawn < Order of matrix

5 < 6

$\therefore$ sol^n nt optimum.

∴ Gr I Gr II Gr III

	P	Q	R	S	T	U
A	13	15	6	1	0	11
B	1	2	12	0	1	7
C	12	0	9	5	1	8
D	1	0	2	6	0	1
E	7	9	0	4	5	0
F	0	4	7	8	5	8

I II

5 < 6

∴ No optimum soln.

∴ Gr I Gr II Gr III

	P	Q	R	S	T	U
A	12	15	5	0	0	10
B	1	3	12	0	2	7
C	11	0	8	4	1	7
D	0	0	1	5	0	0
E	7	10	0	4	6	0
F	0	5	7	8	6	8

$\boxed{6 = 6}$ ∴ optimum soln

	P	Q	R	S	T	U
A	12	15	5	$\cancel{0}$	$\boxed{0}$	10
B	1	3	12	$\boxed{0}$	2	7
C	11	$\boxed{0}$	8	4	1	7
D	$\cancel{0}$	$\cancel{0}$	1	5	$\cancel{0}$	$\boxed{0}$
E	7	10	$\boxed{0}$	4	6	$\cancel{0}$
F	$\boxed{0}$	5	7	8	6	8

A	→	T	69
B	→	S	67
C	→	Q	65
D	→	U	73
E	→	R	71
F	→	P	70
		*	415 Minimum.

Maximizaⁿ problem.

	A	B	C	
max →	$\boxed{20}$	18.	14 ←	min.
Profit maximum ↓	forego			convert.
Opp. cost opportunity cost	0	2	6.	
	↑ min		↑ max	

Maximization Problem :-

Ob

Profit in ₹

	A	B	C	D
P	8	7	6	9
Q	12	10	12	8
R	10	9	$\boxed{14}$	11
S	6	5	7	13

Sol^n → * Given is square matrix.
* as Profit is given
∴ Prob is Maximiza^n
∴ convert into minimiza^n.

Largest no = $\boxed{14}$
∴ subtract every no from 14

A	B	C	D
(14−8)	(14−7)	→	
(14−12)			

Opportunity Cost Matrix

⇒ ∴

	A	B	C	D
P	6	7	8	5
Q	2	4	2	6
R	4	5	0	3
S	8	9	7	1

↳ converted problem
solve normally

Rows

	A	B	C	D
P	1	2	3	0
Q	0	2	0	4
R	4	5	0	3
S	7	8	6	0

Columns

	A	B	C	D	
P	1	0	3	0	II
Q	0	0	0	4	I
R	4	3	0	3	
S	7	6	6	0	
			III	IV	

4 = 4 ⇒ optimum solution

	A	B	C	D
P	1	$\boxed{0}$	3	⊗
Q	$\boxed{0}$	⊗	⊗	4
R	4	3	$\boxed{0}$	3
S	7	6	6	$\boxed{0}$

P → B ⇒ 7
Q → A ⇒ 12
R → C ⇒ 14
S → D ⇒ 13

Total = 46

Maximum Profit in Rs = 46

Prob The Mgmt has to assign 5 salesmen to 5 diff regions. The expected sales of these salesmen in the regions are given below. Suggest the optimum assignment so as to maximize the sales.

Sales in ₹'000

Regions

Salesmen	V	W	X	Y	Z
P	50	58	64	67	65
Q	57	59	60	69	66
R	59	61	65	66	64
S	60	63	66	69	68
T	62	65	68	$\boxed{70}$	62

Sol^n ⇒ The given matrix is a square matrix

Convert sales matrix into opportunity cost matrix

i.e. Maximiza^n to Minimiza^n

∴ Lagest Number is $\boxed{70}$

⟹ ∴ New opp. cost Matrix is

	V	W	X	Y	Z
P	20	12	6	3	5
Q	13	11	10	1	4
R	11	9	5	4	6
S	10	7	4	1	2
T	8	5	2	0	8

* Imp * to find whether given prob is Min / Max Assign.

⟹ Minimize
* by default (nothing mentioned)
1) cost
2) Losses
3) time

* $\boxed{\text{Theory}}$

⟹ Maximize
1) Profit
2) Sales
3) Revenue
4) Efficiency
5) Runs.

∴ Rows r reduced.

	V	W	X	Y	Z
P	17	9	3	0	2
Q	12	10	9	0	3
R	7	5	1	0	2
S	9	6	3	0	1
T	8	5	2	0	8

Columns are reduced

	V	W	X	Y	Z
P	10	4	2	0	1
Q	5	5	8	0	2
R	0	0	0	0	1
S	2	1	2	0	0
T	1	0	1	0	7

∴ 4 < 5

∴ Soln not optimum.

∴ Gr I, Gr II, Gr III

	V	W	X	Y	Z
P	9	3	1	0	0
Q	4	4	7	0	1
R	0	0	0	1	1
S	2	1	2	1	0
T	1	0	1	1	7

4 < 5 ∴ solⁿ nt optimium.

Again Gr I, Gr II, Gr III

	V	W	X	Y	Z
P	8	2	0	0	0
Q	3	3	6	0	1
R	0	0	0	2	2
S	1	0	1	1	0
T	1	0	1	2	8

as 5 = 5 ∴ solⁿ optimum.

∴

			V	W	X	Y	Z
		P	8	2	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$
64	~~4~~	✓ Q	3	3	6	$\boxed{0}$	1
69	~~9~~	R	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	2	2
59	~~7~~	✓ S	1	$\cancel{0}$	1	1	$\boxed{0}$
68	~~7~~	T	1	$\boxed{0}$	1	2	8
65	~~5~~						

325 P → X Q → Y R → V S → Z T → W

64 69 59 68 65

$\boxed{325}$

Maximum Sale is 325000 ₹

Prob

Profit in ₹

Machines

Jobs	A	B	C	D
W	105	111	114	107
X	107	118	112	100
Y	101	112	108	113
Z	99	116	101	107

Sol^n ⇒ Given is square matrix

⇒ Given is Maximiza^n problem

Convert into opp. cost matrix

	A	B	C	D
W	13	7	4	11
X	11	0	6	18
Y	17	6	10	5
Z	19	2	17	11

Rows are reduced

	A	B	C	D
W	9	3	0	7
X	11	0	6	18
Y	12	1	5	0
Z	17	0	15	9

Columns are reduced

	A	B	C
W	~~7~~0	3	0
X	~~3~~2	0	[6]
Y	~~0~~3		5
Z	~~15~~8	0	15

~~3 < 4 ∴ nt opti~~

	A	B	C	D
W	~~7~~	~~9~~	~~[0]~~	~~7~~
X	~~3~~	~~0~~	~~0~~	~~12~~
Y	~~[0]~~	~~17~~	~~5~~	~~0~~
Z	~~9~~	~~0~~	~~9~~	~~3~~

~~4 = 4 ∴ optimum~~

0	3	0	7
2	0	6	18
3	1	5	0
8	0	15	9

3 < 4 ∴ nt optir

		A	B	C	D
W	P	$\cancel{0}$	5	$\boxed{0}$	9
X	Q	$\boxed{0}$	$\cancel{0}$	4	18
Y	R	1	1	3	$\boxed{0}$
Z	S	6	$\boxed{0}$	13	9

4 = 4

W P	→	C	114
X Q	→	A	107
Y R	→	D	113
Z S	→	B	116
			450

∴ Maximum Profit = 450 ₹

Prob Solve the Assignment Prob.

	P	Q	R	S	T
A	42	51	47	58	49
B	43	52	48	59	51
C	48	55	50	56	53
D	50	54	49	57	55
E	46	53	46	58	57

$Sol^n \Rightarrow$ Prob. is default $\therefore$ Minimiza^n.

Rows

	P	Q	R	S	T
A	0	9	5	16	7
B	0	9	5	16	8
C	0	7	2	8	5
D	1	5	0	8	6
E	0	7	0	12	11

Columns

	P	Q	R	S	T
A	0	4	5	8	$\boxed{2}$
B	0	4	5	8	3
C	0	2	2	0	0
D	1	0	0	0	1
E	0	2	0	4	6

4 < 5
not optim

	P	Q	R	S	T
A	~~0~~	2	3	6	$\boxed{0}$
B	$\boxed{0}$	2	3	6	1
C	2	2	2	$\boxed{0}$	~~0~~
D	3	$\boxed{0}$	~~0~~	~~0~~	1
E	2	2	$\boxed{0}$	4	6

49
43
56
54
46
248

5 = 5 $\therefore$ optimum.

$\therefore$ Optimum Sol^n is 248

Prob

Revenue in ₹'00

	A	B	C	D	E
P	98	88	87	85	99
Q	95	87	89	84	100
R	93	82	92	82	95
S	91	80	93	81	96
T	90	81	94	80	94

Soln →

Convert into opp. cost Matrix

	A	B	C	D	E
P	2	12	13	15	1
Q	5	13	11	16	0
R	7	18	8	18	5
S	9	20	7	19	4
T	10	19	6	20	6

⇓

Rows

1	11	12	14	0
5	13	11	16	0
2	13	3	13	0
5	16	3	15	0
4	13	0	14	0

⟹

Columns

0	0	12	1	0
4	2	11	3	0
1	2	3	0	0
4	5	3	2	0
3	2	0	1	0

$\boxed{4 < 5}$

$\boxed{0}$	$\cancel{0}$	12	1	2
2	$\boxed{0}$	9	1	$\cancel{0}$
1	2	3	$\boxed{0}$	2
2	3	1	$\cancel{0}$	$\boxed{0}$
3	2	$\boxed{0}$	1	2

5 = 5

98
87
82
96
94

45,700

Maximize the profit

Prob

Sales in ₹

	P	Q	R	S
A	48	52	51	55
B	47	56	50	58
C	49	54	49	55
D	50	57	53	57

cost in ₹

	P	Q	R	S
A	38	42	44	
B	40	44	46	
C	42	45	47	
D	45	49	48	

Sol^n ⟹ as Profit = Sales − Cost

∴ Sales Matrix − Cost Matrix = Profit Matri

⇓

	P	Q	R	S
A	10	10	7	6
B	7	12	4	8
C	7	9	2	3
D	5	8	5	6

⟹ Opp cost Matri.

2	2	5	6
5	0	8	4
5	3	10	9
7	4	7	6

⇓

Rows [illegible]

0	0	3	4
5	0	8	4
2	0	7	6
3	0	3	2

⟸ columns

0	0	0	2
5	0	5	$\boxed{2}$
2	0	4	4
3	0	0	0

3 < 4

⇓

~~0~~	2	$\boxed{0}$	2
3	$\boxed{0}$	3	~~0~~
$\boxed{0}$	~~0~~	2	2
3	2	~~0~~	$\boxed{0}$

4 = 4.

* Precaution *

~~51~~	−	7
~~56~~	−	12
~~49~~	−	7
~~57~~		6
~~213~~		$\boxed{32}$

18
14
32

* Consider Profit Matrix for final ANS. as profit value is asked.

* Rectangular Matrix *

Prob Solve the assignment problem

	P	Q	R	S	T
A	60	65	68	65	60
B	62	59	67	60	62
C	65	63	64	61	65
D	68	64	66	63	61

⇒ The given matrix is not square matrix
∴ one DUMMY ROW / column is added
Here dummy row is added.

	P	Q	R	S	T
A	60	65	68	65	60
B	62	59	67	60	62
C	65	63	64	61	65
D	68	64	66	63	61
Dummy E	0	0	0	0	0

⇓

Rows	P	Q	R	S	T
A	$\boxed{0}$	5	8	5	~~0~~
B	3	$\boxed{0}$	8	1	3
C	4	2	3	$\boxed{0}$	4
D	7	3	5	2	$\boxed{0}$
DUM	~~0~~	~~0~~	$\boxed{0}$	~~0~~	~~0~~

No need to reduce Column.

A → P	60
B → Q	59
C → S	61
D → T	61
DUM → R	0
	241

Prob

Time in hr

Jobs.

		W	X	Y	Z
Workers	A	25	33	28	35
	B	27	36	29	32
	C	29	30	35	27
	D	32	29	36	26
	E	30	27	31	35

— Which worker sits ideale?

Sol^n ⇒ Given matrix is <u>not</u> square matrix

∴ Add <u>DUMMY COLUMN</u>

	W	X	Y	Z	DUM
A	25	33	28	35	0
B	27	36	29	32	0
C	29	30	35	27	0
D	32	29	36	26	0
E	30	27	31	35	0

No need to reduce Rows

column

$$\begin{matrix} 0 & 6 & 0 & 9 & 0 \\ 2 & 9 & 1 & 6 & 0 \\ 4 & 3 & 7 & 1 & 0 \\ 7 & 2 & 8 & 0 & 0 \\ 5 & 0 & 3 & 9 & 0 \end{matrix} \Rightarrow \begin{matrix} \boxed{0} & 6 & \cancel{0} & 9 \\ 1 & 8 & \boxed{0} & 5 \\ 3 & 2 & 6 & \cancel{0} \\ 7 & 2 & 8 & \boxed{0} \\ 5 & \boxed{0} & 3 & 9 \end{matrix}$$

$4 < 5$

25	A → W	
29	B → Y	4
0	C → DUMMY	∴ C is idle.
26	D → Z	1
27	E → X	2
107		

ANS = Worker C is sitting idle.

Total Minimum time req^d = 107

Prob

The Captain of cricket team has to assign 5 batting positions to 5 batsmen to avg. Runs score by each batsman at these pos^n are as given below. The batsmen at remaining pos^n are expected to score 120 run on avg. Make the assignment so that the avg score of the team is maximum.

Batting pos^n

	III	IV	V	VI	VII
A	40	40	35	25	50
B	42	30	16	25	27
C	50	48	40	60	50
D	20	19	20	18	25
E	28	60	59	55	53

∵ Avg Score of remaining batsmen = 120

⇒ The Runs score must be Maximum

∴ Convert Runs matrix to opp. cost matrix

	III	IV	V	VI	VII
A	20	20	25	35	10
B	18	30	44	35	33
C	10	12	20	0	10
D	40	41	40	42	35
E	2	0	1	5	7

Rows ↓ columns ↓

10	10	15	25	0	→	10
0	12	26	17	15		0
10	12	20	0	10		10
5	6	5	7	0		5
2	0	1	5	7		2

same

$$
\begin{array}{ccccc}
10 & 10 & 15 & 25 & 0 \\
0 & 12 & 25 & 17 & 15 \\
10 & 12 & 19 & 0 & 10 \\
5 & 6 & 4 & 7 & 0 \\
2 & 0 & 0 & 5 & 7
\end{array}
$$

$4 < 5$

$\therefore$ nt opt sol^n

$$
\begin{array}{ccccc}
10 & 6 & 10 & 25 & \boxed{0} \\
\boxed{0} & 8 & 21 & 17 & 15 \\
\cancel{0} & 8 & 15 & \boxed{0} & 10 \\
5 & 2 & 0 & 7 & \cancel{0} \\
6 & \boxed{0} & 0 & 9 & 11
\end{array}
$$

$5 = 5$

$\therefore$ optimum so

$A \rightarrow \text{VII}$ $B \rightarrow \text{III}$ $C \rightarrow \text{VI}$ $D \rightarrow \text{V}$ $E \rightarrow \text{IV}$

50   42   60   20   60

$= 232$ Max score of team.

$+ 120$

352 = total score

Prob The following matrix is daily production of 3 products A, B, C on m/c W, X, Y, Z. Solve assignment problem & which m/c idle?

Daily Production

M/C

	W	X	Y	Z
A	60	59	67	58
B	64	60	62	57
C	62	65	58	68

$\Rightarrow$ Opp. cost matrix

	W	X	Y	Z
A	8	9	1	10
B	4	8	6	11
C	6	3	10	0
DUM	0	0	0	0

he given matrix is Rect. Matrix

Rows r reduced.

	W	X	Y	Z
A	7	8	0	9
B	0	4	2	7
C	6	3	10	0
DUM	0	0	0	0

$\Rightarrow$ columns r reducd.

	W	X	Y	Z
A	7	8	$\boxed{0}$	9
B	$\boxed{0}$	4	2	7
C	6	3	10	$\boxed{0}$
DUM	$\cancel{0}$	$\boxed{0}$	0	$\cancel{0}$

The Maximum zero is coverd

$\therefore$ The Sol^n is optimum.

A $\rightarrow$	Y	67
B $\rightarrow$	W	64
C $\rightarrow$	Z	68
DUM $\rightarrow$	X	0
		199

$\Rightarrow$ X m/c is idle.

Prob

Sales in ₹'000

Counters.

	I	II	III	IV
A	5	13	11	9
B	7	14	10	12
C	9	8	16	11
D	8	12	13	10
E	10	13	14	12

The Given is Rect. Matrix.
Also The prob is Maximizaⁿ

∴ 1st convert into Minimizaⁿ & then add DUMMY column } IMP.

	I	II	III	IV
A	11	3	5	7
B	9	2	6	4
C	7	8	0	5
D	8	4	3	6
E	6	3	2	4

⟹

I	II	III	IV	DUM
11	3	5	7	0
9	2	6	4	0
7	8	0	5	0
8	4	3	6	0
6	3	2	4	0

Rows

	I	II	III	IV	DUM
A	5	1	5	3	0
B	3	0	6	0	0
C	1	6	0	1	0
D	2	2	3	2	0
E	0	1	2	0	0

Columns no need.

4 < 5 ∴ nt optimum

→

	I	II	III	IV	DUM
A	4	$\boxed{0}$	5	2	$\cancel{0}$
B	3	$\cancel{0}$	7	$\boxed{0}$	1
C	$\cancel{0}$	5	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$
D	1	1	3	1	$\boxed{0}$
E	$\boxed{0}$	1	3	$\cancel{0}$	1

5 = 5 ∴ opt solⁿ.

A → II		13
B → IV		12
C → III		16
D →	DUM	0
E →	I	10

₹ 51,000 Max. profit

Prob

Route

Prohibited ~~Rute~~ condition

Time in hr

workers

	A	B	C	D
W	32	35	32	27
X	27	M	35	28
Y	34	31	M	27
Z	38	30	29	32

worker B can't be assigned to job X

—"— C —"— Y

Rows

	A	B	C	D
W	5	8	5	0
X	0	M	8	1
Y	7	4	M	0
Z	9	1	0	3

Columns

A	B	C	D
5	7	5	0
0	M	8	1
7	3	M	0
9	0	0	3

3 < 4 ∴ not opt

	A	B	C	D
W	5	4	2	$\boxed{0}$
X	$\boxed{0}$	M	5	1
Y	7	$\boxed{0}$	M	$\cancel{0}$
Z	12	$\cancel{0}$	$\boxed{0}$	6

4 = 4 ∴ opt soln.

W → D 27

X → A 27

Y → B 31

Z → C 29

114 hr.

Prob Prohibited Route Cond^n

	A	B	C	D	E
M1	9	11	15	10	11
M2	12	9	M	10	9
M3	M	11	14	11	7
M4	14	8	12	7	8
DUM	0	0	0	0	0

Rows

0	2	6	1	2
3	0	M	1	0
M	4	7	4	0
7	1	5	0	1
0	0	0	0	0

$\Rightarrow$

Column

$\boxed{0}$	2	6	1	2
3	$\boxed{0}$	M	1	0
M	4	7	4	$\boxed{0}$
7	1	5	$\boxed{0}$	1
0	0	$\boxed{0}$	0	0

5 = 5

M1	$\rightarrow$	A	9
M2	$\rightarrow$	B	9
M3	$\rightarrow$	E	7
M4	$\rightarrow$	D	7
DUM	$\rightarrow$	C	0
			32

Corporation decided to give one contractor only one road for repairs.

A city corporation has decided to carry out road repairs on four main streets of the city. The ~~city~~ state govt has agreed to make a special grant of Rs 50 lakhs, (if exceeds 50 Lakhs) ~~towards the cost with minimum lowest cost~~

There are five contractors who sent their bids for each road. & the work they all assured in quickest time.

The Cost of Road repairs in lakhs is given below →

	Road 1	Road 2	Road 3	Road 4
C_1	9	14	19	15
C_2	7	17	20	19
C_3	9	18	21	16
C_4	10	12	18	19
C_5	10	15	21	16

Find 1) What [illegible]

2) Which of the five contractor is likely to get unsuccessful in the bid?

3) How much money required from state govt?

	R_1	R_2	R_3	R_4	DUM ~~R_5~~
C_1	2	2	1	0	0
C_2	0	5	2	4	0
C_3	2	6	3	1	0
C_4	3	0	0	4	0
C_5	3	3	3	1	0

	R_1	R_2	R_3	R_4	DUM ~~R_5~~
⇒ C_1	1	1	0	0	0
C_2	0	5	2	5	1
C_3	1	5	2	1	0
C_4	3	0	0	5	1
C_5	2	2	2	1	0

	R_1	R_2	R_3	R_4	DUM
C_1	1	1	0	0	0
C_2	0	5	2	5	1
C_3	1	5	2	1	0
C_4	3	0	0	5	1
C_5	2	2	2	1	0

$\Rightarrow$

R_1	R_2	R_3	R_4	DUM
1	1	0	0	1
0	5	2	5	2
0	4	1	0	0
3	0	0	5	2
1	1	1	0	0

	R_1	R_2	R_3	R_4	DUM
C_1	1	1	$\boxed{0}$	$\cancel{0}$	1
C_2	$\boxed{0}$	5	2	5	2
C_3	$\cancel{0}$	4	1	$\boxed{0}$	$\cancel{0}$
C_4	3	$\boxed{0}$	$\cancel{0}$	5	2
C_5	1	1	1	$\cancel{0}$	$\boxed{0}$

$\longrightarrow$ multiple solution

$\cancel{0}$	$\boxed{0}$
5	2
$\boxed{0}$	$\cancel{0}$

$C_1 \rightarrow$ ~~19~~ $R_3 = 19$
$C_2 \rightarrow$ ~~7~~ $R_1 = 7$
$C_3 \rightarrow$ ~~16~~ R_4 (16
$C_4 \rightarrow$ ~~12~~ $R_2 = 12$
$C_5 \rightarrow$ DUM $= 0$

cost for each Road in (Lakhs)

$C_1 \rightarrow R_3 = 19$ (Lakhs)
$C_2 \rightarrow R_1 = 7$
$C_4 \rightarrow R_2 = 12$
$C_5 \rightarrow R_4 = 16$

total 49

$\Longrightarrow$ Either C_3 or C_5 can be idle

$\Longrightarrow$ Total cost is ~~49~~ 54 lakhs
so no additional money requird from State govt

Prob

There are 4 salesmen A, B, C & D. The management has to assign these salesmen to four region W, X, Y & Z. Due to diff η the expected sale for salesmen are in the ratio 3 : 4 : 2 : 1

The total expected sales for the four regions are Rs. 140K, 160K, 200K, 240 respectively for regions W, X, Y & Z.

		Total Sales
Region	W	140,000
	X	160,000
	Y	200,000
	Z	240,000

as A : B : C : D = 3 + 4 + 2 + 1 = 10

$\therefore$ A : B : C : D = 3 : 4 : 2 : 1

= 3 × 14K : 4 × 14K : 2 × 14K : 1 × 14

= 42K : 56K : 28K : 14

	A	B	C	D	in (₹'000)
Region W	42	56	28	14	

Similarly

Region X	48	64	32	16
Region Y	60	80	40	20
Region Z	72	96	48	24

Next page ⟶

Ex sales in ₹'000

Salesmen

	A	B	C	D
W	42	56	28	14
X	48	64	32	16
Y	60	80	40	20
Z	72	96	48	24

~~a~~ as no. are for Sales

∴ Maximize the sale

∴ prob of Maximizaⁿ

∴ convert sales matrix to opportunity cost matrix

	A	B	C	D
W	54	40	[illegible]	82
X	48	[illegible]	[illegible]	[illegible]
Y	36	16	56	76
Z	24	0	48	72

⇓

Rows r reduced

	A	B	C	D
W	14	0	28	42
X	16	0	32	48
Y	20	0	40	60
Z	24	0	48	72

⟹

Columns r reduced

	A	B	C	D
W	0	0	0	0
X	2	0	4	6
Y	6	0	12	18
Z	10	0	20	30

2 < 3 ⟹

∴ not optm

	A	B	C	D
W	0	2	0	0
X	0	0	2	4
Y	4	0	10	16
Z	8	0	18	28

$3 < 4$

∴ nt optm.

$\Rightarrow$

	A	B	C	D
W	0	6	0	0
X	0	4	2	4
Y	0	0	6	12
Z	4	0	14	24

$3 < 4$ ∴ nt opt

$\Downarrow$

	A	B	C	D
W	2	8	0	$\boxed{0}$
X	0	4	$\boxed{0}$	2
Y	$\boxed{0}$	0	4	10
Z	4	$\boxed{0}$	12	22

[illegible] = 4 [illegible] opt [illegible]

[illegible] [illegible] → A

[illegible] [illegible]

32 X → C

14 W → D

202000 = Max expected Sales

ANS.

basic

Xportation

Minimizaⁿ Technique
Equilibrium

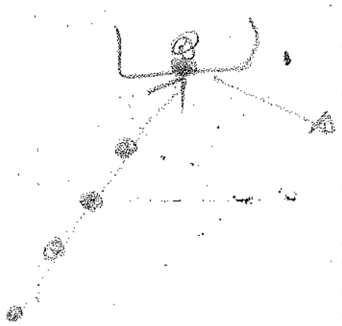
Supply = Demand

plants — Warehouse, Dealer

1st & last column r related

	Dealers			
Plants	X	Y	Z	$\boxed{\text{Supply}}$ (capacity)
A				
B				
C				
D				

1st & last rows r related



Game Theory :-

Game : - description of situ^n where 2/more parties with conflicting interests are involved & each want to take best decisions for himself.

player.

Action str.

Payoff = When game played, then outcome of game is called payoff.

Game Theory (with SADDLE pt)

Prob There r 2 person playing game A & B.
A has 3 actions A_1 A_2 A_3
B has 4 actions B_1 B_2 B_3 B_4.

Following is a Payoff Matrix for A

		Player B			
		B_1	B_2	B_3	B_4
	A_1	9	7	3	9
player A	A_2	9	10	5	6
	A_3	8	6	4	6

Note:- When A player takes action A_1
& B player plays action B_1
then player A wins Rs 9 $\Rightarrow$ (as Payoff Matrix for A).
& B losses Rs 9

Note:- Matrix = Payoff Profit Matrix for A
Loss —— " —— B

(i) to find best action for player A
(ii) —— " —— " —— B
(iii) ~~Now we can observe~~
observe for saddle pt.

(i) To find best action for player A.

⇝ How player A will think?

- A_1 if take action A_1 (Row 1)
  then B may take B_1 B_2 B_3 B_4
  with corrosponding wining payoff are } 9, 7, 3, 9
  & minimum gain is 3 Rs.

- Row 2; for A_2 min = 5 (9, 10, 5, 6)
- Row 3 for A_3 min = 4 (8, 6, 4 6)

∴ Choose Max from min
i.e. 3, 5, 4
∴ 5 = Max from Min
∴ 5 = Maximin.
∴ Best action for A is A_2
& corrosponding value for maximin is 5

(ii) to find best action for player B.

Find Minimax.
i.e. 1st Maximum values from each Column & then min from selected

→ B tries to minimise his maximum losses.

	B_1	B_2	B_3	B_4
A_1	9	7	3	9
A_2	9	10	5	6
A_3	8	6	4	6
Maximum	9	10	5	9

minimax 5

∴ Best action for B is B_3
& corrosponding value for minimax is 5

(iii) Observe or Check for SADDLE pt.

if minmax value = maximin value

then SADDLE pt exists.

here 5 = 5 ∴ saddle pt exist.

i.e. (A_2, B_3)

& value of game is 5.

1. saddle
2. 2 x 2 matrix
3. domianance
4. —n— 3x3 matrix.

Prob 2

	B_1	B_2	B_3	B_4	min	maximin
A_1	1	7	3	4	1	
A_2	5	6	4	5	4	4
A_3	7	2	0	3	0	
max	7	7	4	5		
minimax			4			

(i) to find best action for player A.

∴ find Min value from each row

∴ 1, 4, 0

& maximin = 4. ∴ best action A_2

(ii) to find best action for player B.

∴ find max values from each column.

∴ 7, 7, 4, 5

∴ minimax = 4 Best action B_3

(iii) maximin $\overset{?}{=}$ minimax

4 = 4 yes.

∴ Saddle pt exists i.e. (A_2, B_3)

& value of game is 4

Prob 3

Solve the following game

		B's player			
		I	II	III	min	maxmin
A player	I	0	0	5	0	
	II	2	1	2	1	1
	III	-3	0	-3	-3	
	max	2	1	5		
	minmax		1			

∴ minmax = maxmin = 1

∴ Saddle pt exist (A II B II)

∴ Value of Game = 1.

Prob 4

for the following pay off matrix of firm, Determine the optimal str. for both the firms & value of the game.

	Firm B					min	maxmin
Firm A	3	-1	4	6	7	-1	
	-1	8	2	4	12	-1	
	16	8	6	14	12	6	6
	1	11	-4	2	1	-4	
max	16	11	6	14	12		
minmax			6				

∴ minmax = maxmin = 6

∴ SADDLE pt exist at A3 B3

& value of the game = 6.

2×2 Game
Saddle pt does n't exist.

for given 2×2 game; Profit matrix for A.
1) Check for saddle pt.
2) find probabilities at which each player takes available actions?
3) Find value of the game.

		Player B	
		B_1	B_2
player	A_1	5	1
	A_2	3	4

Sol^n => (1) Check for SADDLE pt.

	B_1	B_2	min	maximin
A_1	5	1	1	3
A_2	3	4	3	
max	5	4		
minimax		4		

∴ maximin value ≠ minimax value
3 ≠ 4
∴ saddle pt doesn't exits

(2) Probabilities. (for player A)
→ Let (P) be Prob. at which ~~pla~~ ~~be~~ A takes action A_1
→ ∴ (1 − P) ——— " ——— " ——— A_2

→ Let Q be ——— " ——— B takes action B_1
∴ (1 − Q) ——— " ——— " ——— B_2

prob.		Q	$(1-Q)$
		B_1	B_2
P	A_1	5	1
$(1-P)$	A_2	3	4

$\therefore$ When A takes action A_1
B may take actions B_1 & B
with corresponding payoff 5 & 1

$\therefore$ If A player takes action A_1; his expected profits are

$$E(A_1) = 5Q + (1-Q)$$

Similarly
If A player takes action A_2;

$$E(A_2) = 3Q + 4(1-Q)$$

* How A will think?
 - for A expected profit for Both actions A_1 & A_2 should be same

$$\therefore \quad E(A_1) = E(A_2)$$

$$\therefore \quad 5Q + (1-Q) = 3Q + 4(1-Q)$$

$$\therefore \quad 5Q + 1 - Q = 3Q + 4 - 4Q$$

$$5Q = 3$$

$$\therefore \quad Q = 3/5$$

$$\therefore \quad 1 - Q = 1 - 3/5 = 2/5$$

$$\therefore \quad Q = \frac{3}{5} \qquad 1 - Q = \frac{2}{5}$$

~~85~~ 17
~~25~~ 5

32
36
1.7
85
25

prob at which both players are playing *

Prob		$3/5$	$2/5$
		B_1	B_2
$1/5$ ~~1/3~~	A_1	5	1
$4/5$ ~~2/3~~	A_2	3	4

Consider player B. → Expected Losses.

$$E(B_1) = 5P + 3(1-P)$$

$$E(B_2) = P + 4(1-P)$$

check problem

How B will think?

B will expect same losses from action B_1 & B_2

$\therefore \quad E(B_1) = E(B_2)$

$\therefore \quad 5P + 3(1-P) = P + 4(1-P)$

$\therefore \quad 5P + 3 - 3P = P + 4 - 4P$

$\therefore \quad 2P + 3 = 4 - 3P$ here.

$\therefore \quad \cancel{3P} \; 5P = 1$

$P = \frac{1}{\cancel{3}\,5}$ $\quad 1 - P = \cancel{2/3} \; \frac{4}{5}$

$\therefore$ A will take action A_1 with prob. = ~~1/3~~ $1/5$
A —"— A_2 —"— = $4/5$ ~~2/3~~
B —"— B_1 —"— = $3/5$
B —"— B_2 —"— $2/5$
*

(iii) $\boxed{\text{value of the game}}$ * Draw complete

Expected Payoff.

Joint Actions of A & B		Joint Prob.	Payoff	Produce ~~Payoff (Joint prob)~~
A_1	B_1	$\frac{1}{5}\cancel{\frac{1}{3}} \cdot \frac{3}{5} = \frac{3}{25}$	5	$15/25$
A_1	B_2	$\frac{1}{5}\cancel{\frac{1}{3}} \cdot \frac{2}{5} = \frac{2}{25}$	1	$2/25$
A_2	B_1	$\frac{4}{5}\cancel{\frac{2}{3}} \cdot \frac{3}{5} = \cancel{6/15}\; \frac{12}{25}$	3	~~18/15~~ $36/25$
A_2	B_2	$\frac{4}{5}\cancel{\frac{2}{3}} \cdot \frac{2}{5} = \cancel{4/15}\; 8/25$	4	~~16/15~~ ~~2/12~~ $32/25$ +

Expected payoff Total = ~~X~~ $\boxed{\cancel{51/15}}$

$\therefore$ Value of the game = ~~51/15~~ $\frac{15+2+36+32}{25} = \frac{17}{5}$

Prob.

Solve the following Game

	B_1	B_2	B_3	B_4	min	~~maximax~~ maximin
A_1	1	7	3	4	1	
A_2	5	6	4	5	4	4
A_3	7	2	0	3	0	
max	7	7	4	5	4	

~~maxi~~minimax 4

$\therefore$ maximin = minimax as $4 = 4$

$\therefore$ Saddle pt exists at (A_2, B_3)

& value of Game = 4

Prob.

	B_1	B_2
A_1	6	-3
A_2	-3	0

(i) check for saddle pt

	B_1	B_2	min	maximin
A_1	6	-3	-3	-3
A_2	-3	0	-3	
max	6	0		

minimax 0

as $-3 \neq 0$ $\therefore$ saddle pt Nt exists.

(ii) Probabilities

		Q	$(1-Q)$
		A_1	B_2
P	A_1	6	-3
$(1-P)$	A_2	-3	0

Expected Profit for player A is same

$E(A_1) = E(A_2)$

$6Q + (-3)(1-Q) = -3(Q) + 0(1-Q)$

$6Q - 3 + 3Q = -3Q \Rightarrow Q = 3/12 = 1/4$

$\therefore Q = 1/4$ $1 - Q = 3/4$

Expected Loss by player B is same

$$E(B_1) = E(B_2)$$

$$6P + (-3)(1-P) = -3P + 0(1-P)$$

$$6P - 3 + 3P = -3P \Rightarrow 12P = 3$$

$$\therefore P = \frac{3}{12} = \frac{1}{4}$$

$P = 1/4$ $1 - P = 3/4$

∴ Game probability at which both players play is

Prob.		1/4	3/4
		B_1	B_2
1/4	A_1	6	−3
3/4	A_2	−3	0

check problem

(iii) Value of the game.

Joint Action of A & B		Joint Prob.	Payoff	Product
A_1	B_1	$1/4 \cdot 1/4 = 1/16$	6	6/16
A_1	B_2	$1/4 \cdot 3/4 = 3/16$	−3	−9/16
A_2	B_1	$3/4 \cdot 1/4 = 3/16$	−3	−9/16
A_2	B_2	$3/4 \cdot 3/4 = 9/16$	0	0/16 *

∴ Value of the game

$$\frac{6 - 9 - 9 + 0}{16} = \frac{-12}{16} = -\frac{3}{4}$$

Dominance Property.

— Used to find sol^n of the game
↳ in case of pure str. / where saddle pt exists
↳ reduce game from higher to lower order

★ In dominance property, if a particular player if one action dominates the other one, then other one is deleted from the game.

	B_1	B_2	B_3	B_4	B_5	min
A_1	1	3	2	7	4	1
A_2	3	4	1	5	6	1
A_3	6	$\boxed{5}$	7	6	5	5
A_4	2	0	6	3	1	0
max	6	5	7	7	6	

maximin 5

minimax 5

Value of game is 5 $A_3 B_2$

ROWS

we will solve this by Dominance property

for dominance first compare all rows.

A_1 with A_2

$1 \leq 3$, $3 \leq 4$, but $\underline{2 > 1}$ ∴ can't use.

A_1 with A_3

$1 \leq 6$, $3 \leq 5$, $2 \leq 7$, $\underline{7 > 5}$ ∴ cant use.

and so on compare.

A_3 with A_4

$6 \geq 2$ $5 \geq 0$ $7 \geq 6$ $6 \geq 3$ $5 \geq 1$

Small Remove

∴ A_3 is dominating to A_4

& A_4 is dominated by A_3

* For Rows, which Row is dominated by ~~higher~~ other is Removed

∴ Here A_4 is Removed

∴ Remaining Game is.

	B_1	B_2	B_3	B_4	B_5
A_1	1	3	2	7	4
A_2	3	4	1	5	6
A_3	6	5	7	6	5

→ Compare $\boxed{\text{Columns}}$.

Compare B_1 & B_2

$1 \leq 3$ $3 \leq 4$ But $6 > 5$ ∴ can't use

Compare B_1 & B_4

$1 < 7$ $3 \leq 5$ $6 \leq 6$

large delete

∴ B_4 Column is Dominating B_1
& B_1 is dominated by B_4.

* as B is corrosponding to loss.
* for column, Delete Column which is Dominating
i.e. B_4

Remaining Game

	B_1	B_2	B_3	~~B_5~~
A_1	1	3	2	4
A_2	3	4	1	6
A_3	6	5	7	5

Compare Column: B_2, B_5

$3 < 4$ $4 < 6$ $5 \leq 5$

∴ B_5 is Dominating ∴ Remove B_5

Remaing Game

	B_1	B_2	B_3
~~A_1~~	1	3	2
~~A_2~~	3	4	1
A_3	6	5	7

Compare $\boxed{\text{Rows}}$

A_1 & A_3

$1 < 6$ $3 < 5$ $2 < 7$ ∴ A_1 is Dominated by A_3

∴ Remove A_1

A_2 A_3

$3 < 6$ $4 < 5$ $1 < 7$ ∴ A_2 is Dominated by A_3

∴ Remove A_2

$\therefore$ Remaining Matrix is

	B_1	B_2	B_3
A_3	6	5	7

Compare $\boxed{\text{Columns}}$

— B_1 & B_2 ; $6 > 5$

$\therefore$ B_1 is Dominating $\therefore$ Remove B_1

— B_2 & B_3 ; $5 < 7$

$\therefore$ B_3 is dominating $\therefore$ Remove B_3

$\therefore$ Remaining is

	B_2
A_3	5

$\therefore$ Best Action for player A is A_3

B is B_2

$\therefore$ Value of the game is $\boxed{5}$

2b. Solve the game (using dominance property)

	B_1	B_2	B_3	min	maximin
A_1	8	-3	7	-3	
A_2	6	-4	5	-4	-3
A_3	-2	2	-3	-3	
max	8	2	7		
minimax		2			

∴ There is no saddle pt exist

Compare Rows →

A_1 & A_2

$8 > 6 \quad -3 > -4 \quad 7 > 5$

∴ A_1 is Dominating & A_2 is Dominated ✓ } Remove A_2

A_1 & A_3

$8 > -2 \quad -3 < 2$ ∴ can't use

∴ Remaining payoff matrix is

	B_1	B_2	B_3
A_1	8	-3	7
A_3	-2	2	-3

Compare Columns →

B_1 & B_2 – can't use

B_1 & B_3

$8 > 7 \quad -2 > -3$ ∴

B_1 is Dominating ✓ & B_3 is Dominated } Remove B_1

B_2 & B_3 – can't use.

$\therefore$ Remaing payoff matrix is

	B_2	B_3
A_1	-3	7
A_3	2	-3

further with Rows compare can't use dominance property.

$\therefore$ Now use 2x2 matrix w/o saddle pt method

Probabilities		Q	$(1-Q)$
		B_2	B_3
P	A_1	-3	7
$(1-P)$	A_3	2	-3

$\longrightarrow$ for player A ;

Expected profits are same

$\therefore$ $E(A_1) = -3(Q) + 7(1-Q)$

$E(A_3) = 2Q - 3(1-Q)$

are same $\therefore$ $-3Q + 7 - 7Q = 2Q - 3 + 3Q$

$\therefore$ $-15Q = -10$

$\therefore$ $Q = \frac{10}{15} = 2/3$

$\therefore$ $Q = 2/3$ $\quad$ $1 - Q = 1/3$

$\longrightarrow$ for player B,

Expected losses are

$E(B_2) = -3P + 2(1-P)$

$E(B_3) = 7P + -3(1-P)$

are same $\therefore$ $-3p + 2 - 2P = 7p - 3 + 3P$

$\therefore$ $-15p = -5$

$P = 1/3$ $\quad$ $1 - P = 2/3$ $\longrightarrow$

Joint Action of A & B	Joint Prob.	Payoff	Product
A_1 B_2	$1/3 \cdot 2/3 = 2/9$	-3	$-6/9$
A_1 B_3	$1/3 \cdot 1/3 = 1/9$	7	$7/9$
A_3 B_2	$2/3 \cdot 2/3 = 4/9$	2	$8/9$
A_3 B_3	$2/3 \cdot 1/3 = 2/9$	-3	$-6/9$
		Total	$3/9$

$\therefore$ value of Game $= 3/9 = 1/3$

	B_1	B_2	B_3	B_4
A_1	3	2	4	0
A_2	3	4	2	4
A_3	4	2	4.	0
A_4	0	4	0	8

$A_1 \leq A_3$ $\therefore$ Delete A_1

	B_1	B_2	B_3	B_4
A_2	3	4	2	4
A_3	4	2	4	0
A_4	0	4	0	8

$B_1 \geq B_3$

$\therefore$ Delete B_1.

	B_2	B_3	B_4
A_2	4	2	4
A_3	2	4	0
A_4	4	0	8

No further red^n possible th^r normal method.

$\therefore$ use Avg. method.

$\Downarrow$

Try

	B_2 & B_3	B_4
A_1	3	4
A_2	3	0
A_3	2	8

$\Longrightarrow$ N+. possible $\times$ step Delete

Try

	B_2	B_3 & B_4
A_1	4	3
A_2	2	2
A_3	4	4

$B_2 \geq (B_2 \& B_4)$

$\therefore$ Delete B_2.

Now

	B_3	B_4
A_2	2	4
A_3	4	0
A_4	0	8

$\Longrightarrow$

Try

	B_3	B_4
A_2	2	4
A_3 & A_4	2	4

$\therefore$ $A_2 \leq (A_3 \& A_4)$

Delete A_2

Now.

	B_3	B_4
A_3	4	0
A_4	0	8

} no saddle pt.

Solve by 2x2 matrix method.

		(Q) B_3	(1-Q) B_4
(P)	A_3	4	0
(1-P)	A_4	0	8

→ for player A. (expected profit)

$E(A_3) = 4Q + 0(1-Q)$

$E(A_4) = 0Q + 8(1-Q)$

$\therefore 4Q = 8 - 8Q$

$= 12Q = 8 \quad \therefore \quad Q = 8/12 = 2/3$

$\therefore 1-Q = 1/3$

→ for Player B. (expected losses)

$E(B_3) = 4P + 0(1-P)$

$E(B_4) = 0P + 8(1-P)$

$4P = 8 - 8P \Rightarrow P = 2/3 \quad 1-P = 1/3$

		2/3 B_3	1/3 B_4
2/3	A_3	4	0
1/3	A_4	0	8

Joint Actions of A_3 B_3	Joint Prob.	Payoff	Product
A_3 B_3	$2/3 \cdot 2/3 = 4/9$	4	$16/9$
A_3 B_4		0	0
A_4 B_3		0	0
A_4 B_4	$1/3 \cdot 1/3 = 1/9$	8	$8/9$.

(*) Total. $\frac{24}{9} = \frac{8}{3}$

Whr u can go wrong.

$\therefore$ Value of game $= \frac{8}{3}$.

B

A	I	II	III	IV	V	VI
I	~~4~~	~~2~~	~~0~~	~~2~~	~~1~~	~~1~~
II	4	3	1	3	2	2
III	4	3	7	-5	1	2
IV	4	3	4	-1	2	2
V	~~4~~	~~3~~	~~3~~	~~-2~~	~~2~~	~~2~~

Row I, V delete

Column I II VI delete

Remaining

	III	IV	V
II	1	3	2
III	7	-5	1
IV	4	-1	2

	III & IV	V
II	2	2
III	1	1
IV	3/2	2

+

Remain :.

	III	IV
II	1	3
III	7	-5
IV	4	-1

$\Rightarrow$

	III	IV
II & III	4	-1
IV	4	-1

$\therefore$

		4/7	3/7
		III	IV
4/7	II	1	3
3/7	III	7	-5

Solve. $\Rightarrow$ final $V = \frac{13}{7}$

Linear Programming

Graphical Method

solved R.C.

Prob 1) Minimize $z = 6x_1 + 5x_2$

Subject to $4x_1 + x_2 \geq 10$

$2x_1 + 3x_2 \geq 15$

$x_1 \leq 10$

$x_1, x_2 \geq 0$

Sol^n:- **Step I**

Treat each constraint as equality

$\therefore 4x_1 + x_2 = 10$ —— I

$2x_1 + 3x_2 = 15$ —— II

$x_1 = 10$ —— III

Step II

Find two points for each constraints

→ Consider $4x_1 + x_2 = 10$ —— (I)

Put $x_1 = 0$ | Put $x_2 = 0$

$\therefore 4(0) + x_2 = 10$ | $4x_1 + (0) = 10$

$\therefore x_2 = 10$ | $\therefore x_1 = 2.5$

$x_2 = (0, 10)$ | $x_1 = (2.5, 0)$

$\therefore$ We got two points $(0, 10)$ & $(2.5, 0)$

→ Consider $2x_1 + 3x_2 = 15$ —— (II)

Put $x_1 = 0$ | Put $x_2 = 0$

$\therefore 2(0) + 3x_2 = 15$ | $\therefore 2x_1 + 3(0) = 15$

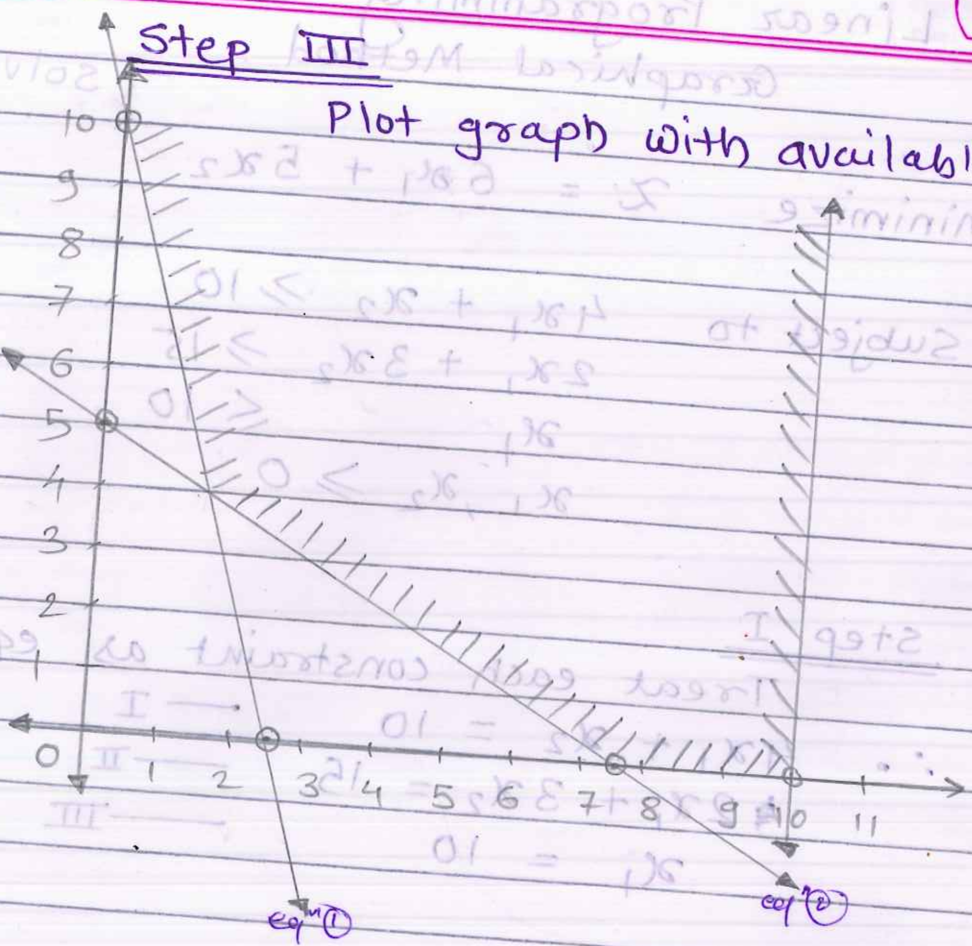
$\therefore x_2 = 5$ | $\therefore x_1 = 7.5$

$\therefore$ We got two points $(0, 5)$ & $(7.5, 0)$

→ Consider $x_1 = 10$ $\therefore$ we get point $(10, 0)$

Step III

Plot graph with available point



Step IV

Draw feasible region

line 1 ⇒ (0, 10) & (2·5, 0) $\geq$ 10

line 2 ⇒ (0, 5) & (7·5, 0) $\geq$ 15

line 3 ⇒ (10, 0) $\leq$ 10

also x_1, x_2 $\geq$ 0

∴ ~~Points for~~ feasible area is 'unbounded polygon' with following points

A (0, 10)

B intersection of line 1 & line 2

C (7·5, 0)

D (10, 0)

Consider $20x_1 + 50x_2 = 300$

Put $x_1 = 0$

$\therefore 20(0) + 50x_2 = 300$

$\therefore 50x_2 = 300$

$\therefore x_2 = 6$

$\therefore (0, 6)$

Put $x_2 = 0$

$\therefore 20x_1 + 50(0) = 300$

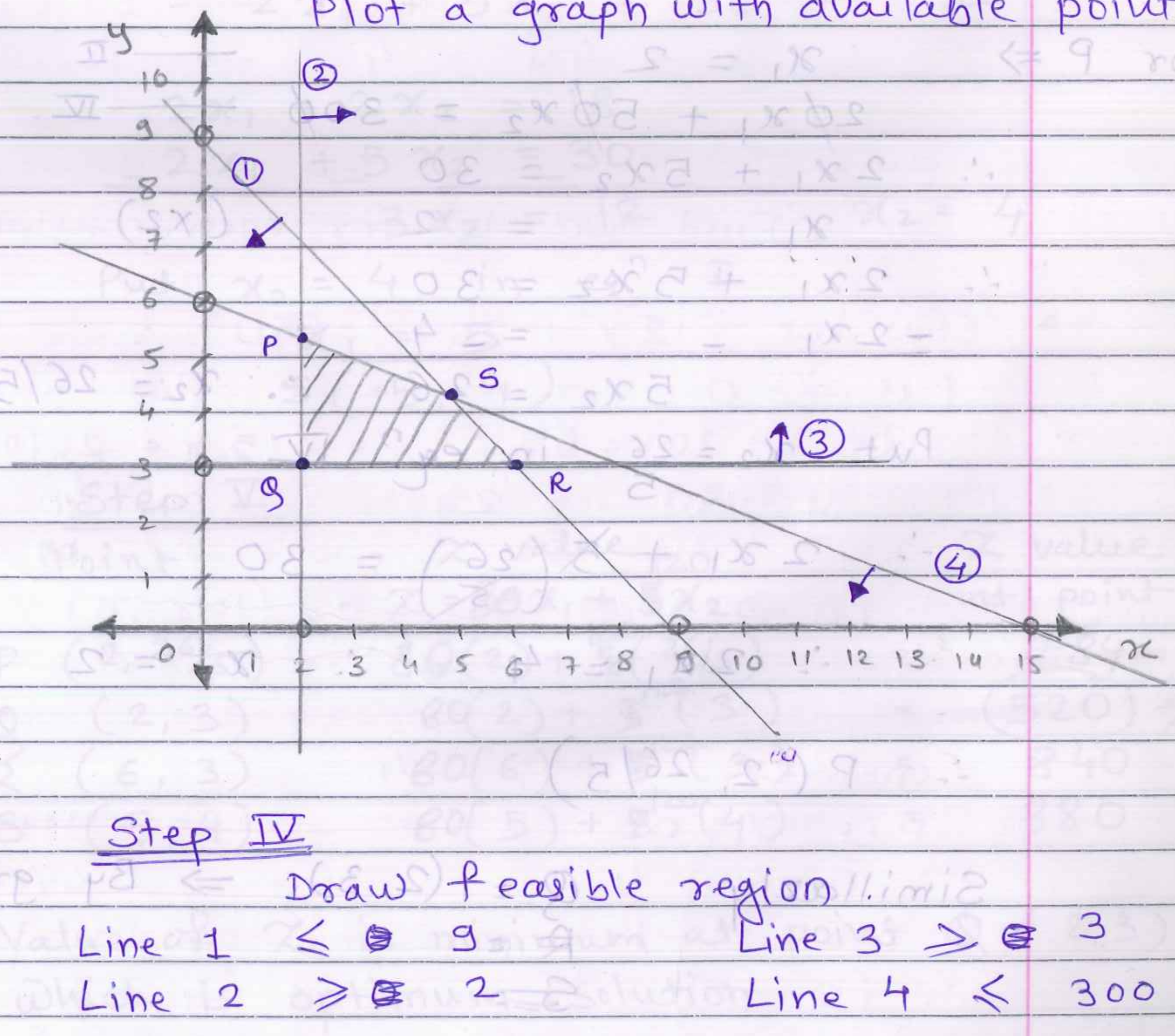
$\therefore 20x_1 = 300$

$\therefore x_1 = 15$

$(15, 0)$

Step III

Plot a graph with available points



Step IV

Draw feasible region

Line 1 $\leq$ 9

Line 2 $\geq$ 2

Line 3 $\geq$ 3

Line 4 $\leq$ 300

$\therefore$ Feasible area is a 'bounded polygon' with following points.

P = Intersection of Lines 4 & 2

Q = —"— 2 & 3 = (2, 3)

R = —"— 1 & 3

S = —"— 1 & 4

Solve respective equations simultaneously

For P ⇒ $x_1 = 2$ —— II

$2\cancel{0}x_1 + 5\cancel{0}x_2 = 30\cancel{0}$ —— IV

$\therefore 2x_1 + 5x_2 = 30$

$x_1 = 2$ $(\times 2)$

$\therefore 2x_1 + 5x_2 = 30$

$-2x_1 \quad = -4$

$5x_2 = 26 \quad \therefore x_2 = 26/5$

Put $x_2 = \frac{26}{5}$ in eq^n IV

$\therefore 2x_1 + \cancel{5}\left(\frac{26}{\cancel{5}}\right) = 30$

$\therefore 2x_1 = 4 \quad \therefore x_1 = 2$

$\therefore P(2, 26/5)$

Simillarly Q = (2, 3) ⇒ By graph

~~R~~ =

~~S~~ =

For R $\Rightarrow$ $x_1 + x_2 = 9$ — I

$$\underline{- \quad -x_2 = -3} \quad \text{— III}$$

$$x_1 = 6$$

$\therefore$ $x_1 = 6$ & $x_2 = 3$

$\therefore$ R (6, 3)

For S $\Rightarrow$ $x_1 + x_2 = 9$ — I $(\times 2)$

$2x_1 + 5x_2 = 30$ — IV

$\therefore$ $2x_1 + 2x_2 = 18$

$$\underline{-2x_1 \; \overset{-}{+} 5x_2 = \overset{-}{}30}$$

$3x_2 = 12$ $\therefore$ $x_2 = 4$

Put $x_2 = 4$ in eqⁿ I

$\therefore$ $x_1 = 5$

$\therefore$ S (5, 4)

Step V

Point	Z value $Z = 60x_1 + 120x_2$		Z value at point
P $(2, 26/5)$	$60(2) + 120(26/5)$	=	784
Q $(2, 3)$	$60(2) + 120(3)$	=	520 *
R $(6, 3)$	$60(6) + 120(3)$	=	840
S $(5, 4)$	$60(5) + 120(4)$	=	880

Value of Z is minimum at point Q (2, 3) which is optimum solution.

$Z_{min} = 520$ at $x_1 = 2$ & $x_2 = 3$

solved RC

[multiple sol^n]

Prob 3) Maximize $z = 3x + 5y$

S.t.

$\cancel{12x + 20}$

$12x + 20y \leq 40$

$x - 4y \leq 4$

$x, y \geq 0$

Sol^n

Step I

Treat each constraint as equalit

$\therefore 12x + 20y = 40$ —— I

$x - 4y = 4$ —— II

Step II

Find two pts. for each constraint

→ Consider $12x + 20y = 40$

Put $x = 0$

$\therefore 12(0) + 20y = 40$

$\therefore y = \frac{40}{20} = 2$

$\therefore (0, 2)$

Put $y = 0$

$12x + 20(0) = 4$

$\therefore x = \frac{40}{12} = \frac{10}{3}$

$\therefore (10/3, 0)$

3.33

→ Consider $x - 4y = 4$

Put $x = 0$

$\therefore 0 - 4y = 4$

$\therefore y = -1$

$\therefore (0, -1)$

Put $y = 0$

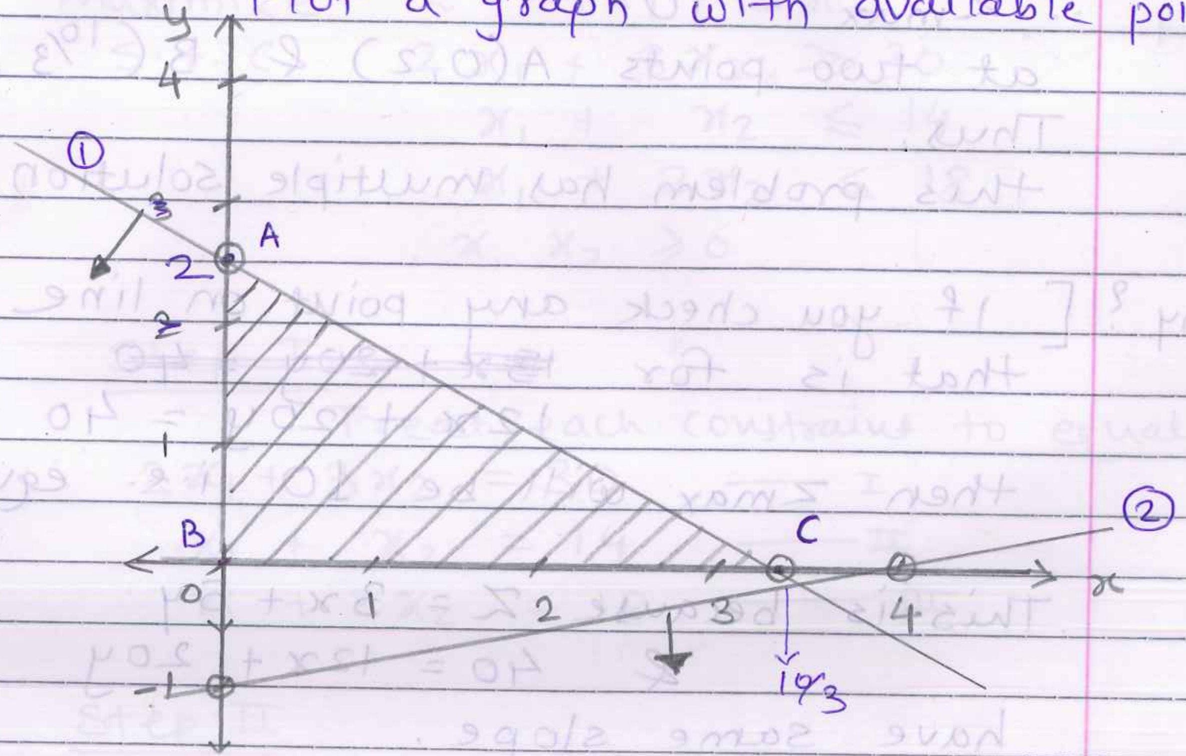
$\therefore x - 4(0) = 4$

$\therefore x = 4$

$\therefore (4, 0)$

Step III

Plot a graph with available points.



Step IV

Draw feasible region.

Line 1 $\leq$ & Line 2 $\leq$ & $x, y \geq 0$

$\therefore$ feasible Region is a bounded triangle with following points.

A (0 , 2)
B (0 , 0)
C (10/3 , 0)

} direct from graph.

Step V

Point	Z value $Z = 3x + 5y$	Z value at point
A (0, 5/2)	$3(0) + 5(\cancel{5}2)$	10
B (0, 0)	$3(0) + 5(0)$	0
C (10/3, 0)	$3(10/3) + 5(0)$	10

& Multiple Solution

$\therefore Z_{max} = 10$
at two points $A(0,2)$ & $B(10/3, 0)$
Thus,
this problem has multiple solution.

Why? [If you check any point on line 1
that is for ~~$15x + 20y = 40$~~
$12x + 20y = 40$
then Zmax will be 10 i.e. equal.

This is because $Z = 3x + 5y$
& $40 = 12x + 20y$
have same slope.

→ Consider $Z = 3x + 5y$
$\therefore 5y = -3x + Z$
$\therefore y = \left(-\frac{3}{5}\right)x + \left(\frac{Z}{5}\right)$
→ slope $= \left(-\frac{3}{5}\right)$

→ Consider $40 = 12x + 20y$
$\cancel{4} \times 10 = \cancel{4}(3x + 5y)$
$\therefore 5y = -3x + 10$
$\therefore y = \left(-\frac{3}{5}\right)x + 2$
→ slope $= \left(-\frac{3}{5}\right)$

unsolved from R.C.

PUMBA - May 06

Prob 4)

Maximize $z = 4x_1 + 2x_2$

s.t.c. $2x_1 + 3x_2 \geq 30$

$x_1 + x_2 \leq 14$

$x_1 + 2x_2 \leq 18$

$x_1, x_2 \geq 0$

Sol^n =

Step I

Treat each constraint to equality

$\therefore 2x_1 + 3x_2 = 30$ —— I

$x_1 + x_2 = 14$ —— II

$x_1 + 2x_2 = 18$ —— III

Step II

Find two pts for each constraint

→ consider $2x_1 + 3x_2 = 30$

Put $x_1 = 0$ Put $x_2 = 0$

$\therefore 0 + 3x_2 = 30$ $\therefore 2x_1 + 0 = 30$

$\therefore x_2 = 10$ $\therefore x_1 = 15$

$\therefore (0, 10)$ $(15, 0)$

→ Consider $x_1 + x_2 = 14$

Put $x_1 = 0$

$\therefore x_2 = 14$

$(0, 14)$ simillarly $(14, 0)$

→ Consider $x_1 + 2x_2 = 18$

Put $x_1 = 0$ Put $x_2 = 0$

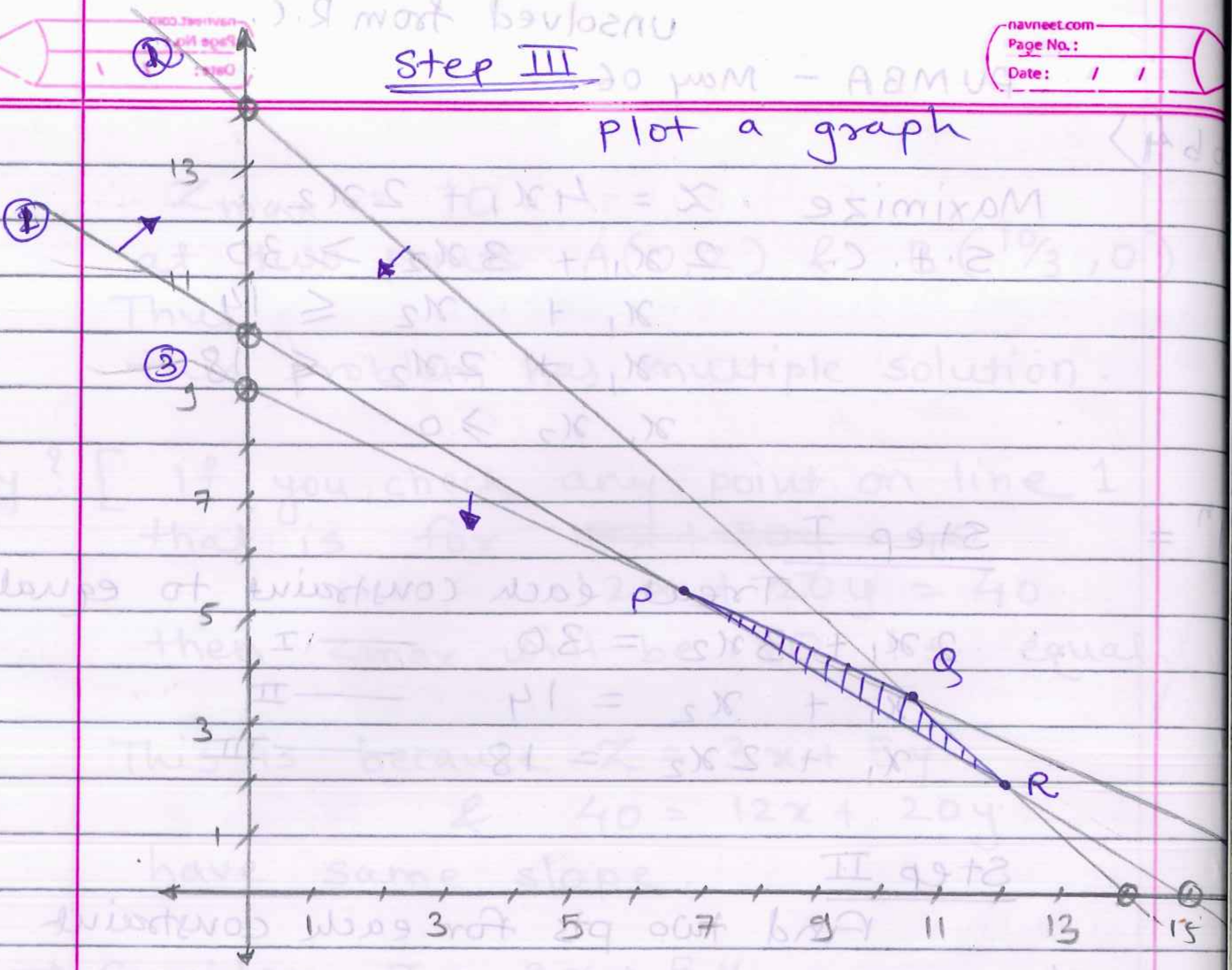
$\therefore 0 + 2x_2 = 18$ $\therefore x_1 + 0 = 18$

$\therefore x_2 = 9$ $\therefore x_1 = 18$

$\therefore (0, 9)$ $(18, 0)$

Step III

Plot a graph



Step IV

Draw feasible region

Line 1 ≥ 30

Line 2 ≤ 14

Line 3 ≤ 18

$\therefore$ feasible region is a ~~un~~ bounded traing

with following points.

P = Intersection of line 1 & 3

Q = ——— " ——— 2 & 3

R = ——— " ——— 1 & 2

Solve respective equations simultaneously

for P $\Rightarrow$

$$\begin{array}{rl} 2x_1 + 3x_2 = 30 & \text{— I} \\ -2x_1 \pm 4x_2 = -36 & \text{— III} \times 2 \\ \hline + x_2 = +6 & \end{array}$$

Put $x_2 = 6$ in eq^n III

$\therefore x_1 + 2(6) = 18$

$\therefore x_1 = 6$

$\therefore P(6, 6)$

for Q $\Rightarrow$

$$\begin{array}{rl} x_1 + x_2 = 14 & \text{— II} \\ -x_1 \pm 2x_2 = -18 & \text{— III} \\ \hline + x_2 = +4 & \end{array}$$

$x_1 = 10$

$\therefore Q(10, 4)$

for R $\Rightarrow$

$$\begin{array}{rl} 2x_1 + 3x_2 = 30 & \text{— I} \\ -2x_1 + -2x_2 = -28 & \text{— II} \times 2 \\ \hline x_2 = 2 & \end{array}$$

Put $x_2 = 2$ in eq^n II $\therefore x_1 = 12$

$\therefore R = (12, 2)$

Step V

Point	Z value $4x_1 + 2x_2$	Z value at point
P (6,6)	4 (6) + 2 (6)	36
Q (10,4)	4 (10) + 2 (4)	48
R (12,2)	4 (12) + 2 (2)	* 52

Value of Z is maximum at pt R (12, 2) which is optimum solution.

$Z_{max} = 52$ at $x_1 = 12$ & $x_2 = 2$

Unsolved R.C.

Problem 5)

Maximize $Z = 6x_1 + 14x_2$

S.t.C. $5x_1 + 4x_2 \geq 60$

$3x_1 + 7x_2 \geq 84$

$x_1 + 2x_2 \geq 18$

$x_1, x_2 \geq 0$

Sol^n = Step I

Treat each constraint as equality

$\therefore 5x_1 + 4x_2 = 60$ ——— I

$3x_1 + 7x_2 = 84$ ——— II

$x_1 + 2x_2 = 18$ ——— III

Step II

Find two pts for each constraint

→ Consider $5x_1 + 4x_2 = 60$

$\therefore$ Put $x_1 = 0$	Put $x_2 = 0$
$\therefore 5(0) + 4x_2 = 60$	$\therefore 5x_1 + 0 = 60$
$\therefore x_2 = 15$	$\therefore x_1 = 12$
$\therefore (0, 15)$	$(12, 0)$

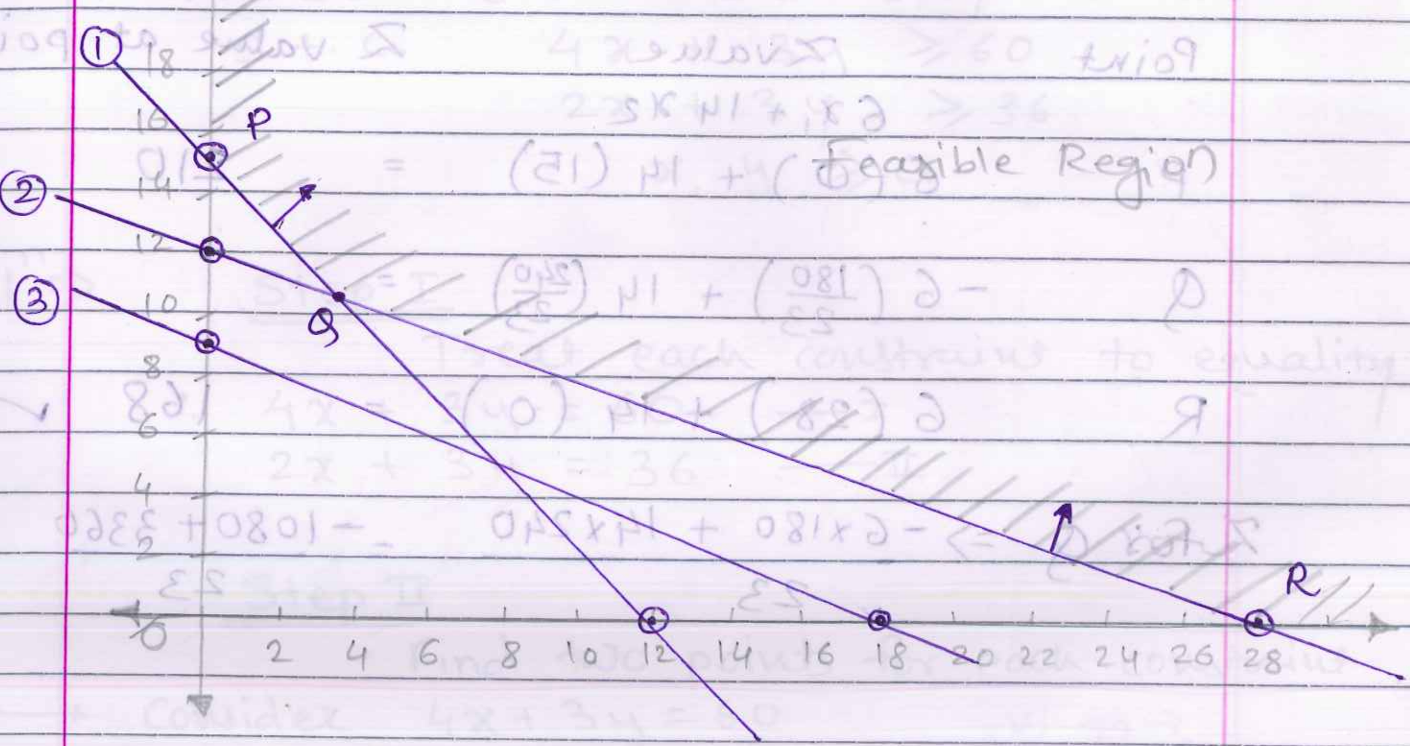
→ Consider $3x_1 + 7x_2 = 84$

Put $x_1 = 0$	Put $x_2 = 0$
$\therefore 3(0) + 7x_2 = 84$	$\therefore 3x_1 + 0 = 84$
$\therefore x_2 = 12$	$\therefore x_1 = 28$
$(0, 12)$	$(28, 0)$

→ Consider $x_1 + 2x_2 = 18$

$\therefore$ Put $x_1 = 0$	Put $x_2 = 0$
$\therefore 0 + 2x_2 = 18$	$\therefore x_1 + 0 = 18$
$\therefore x_2 = 9$	$\therefore x_1 = 18$
$\therefore (0, 9)$	$(18, 0)$

Step III Plot a graph.



Step IV

Draw a feasible region

Feasible region is a unbounded polygon with following points.

P = (0, 15)

Q = intersection of line 1 & line 2 $\Rightarrow \left(\frac{180}{23}, \frac{240}{23}\right)$

R = (28, 0)

$\therefore Q \Rightarrow$

$$5x_1 + 4x_2 = 60 \quad \times 3$$

$$3x_1 + 7x_2 = 84 \quad \times 5$$

$$\therefore 15x_1 + 12x_2 = 180$$

$$15x_1 + 35x_2 = 420$$

$$23x_2 = 240 \quad \therefore x_2 = \frac{240}{23}$$

Put $x_2 = 240/23$ in eq^n I

$$\therefore 5x_1 + \frac{4 \times 240}{23} = 60$$

$$\therefore x_1 = \frac{1}{5}\left(\frac{60 - 960}{23}\right) = \frac{-900}{115} = \frac{180}{23}$$

Step V

Point	Z value $6x_1 + 14x_2$		Z value at P
P	$6(0) + 14(15)$	=	210
Q	$-6\left(\frac{180}{23}\right) + 14\left(\frac{240}{23}\right)$	=	
R	$6(28) + 14(0)$	=	168

Z for Q $\Rightarrow \dfrac{-6\times180 + 14\times240}{23} = \dfrac{-1080 + 3360}{23}$

unsolved from R. C.

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7.53

Prob. 6 >

Mimimize $z = 25x + 30y$

s.t.c. $4x + 3y \geq 60$

$2x + 3y \geq 36$

$x, y \geq 0$

Sol^n =>

Step I

Treat each constraint to equality

$\therefore 4x + 3y = 60$ —— I

$2x + 3y = 36$ —— II

Step II

Find two points for each constraint.

→ Consider $4x + 3y = 60$

Put $x = 0$

$\therefore 4(0) + 3y = 60$

$\therefore y = 20$

$\therefore (0, 20)$

Put $y = 0$

$\therefore 4x + 0 = 60$

$\therefore x = 15$

$(15, 0)$

→ Consider $2x + 3y = 36$

Put $x = 0$

$\therefore 0 + 3y = 36$

$\therefore y = 12$

$(0, 12)$

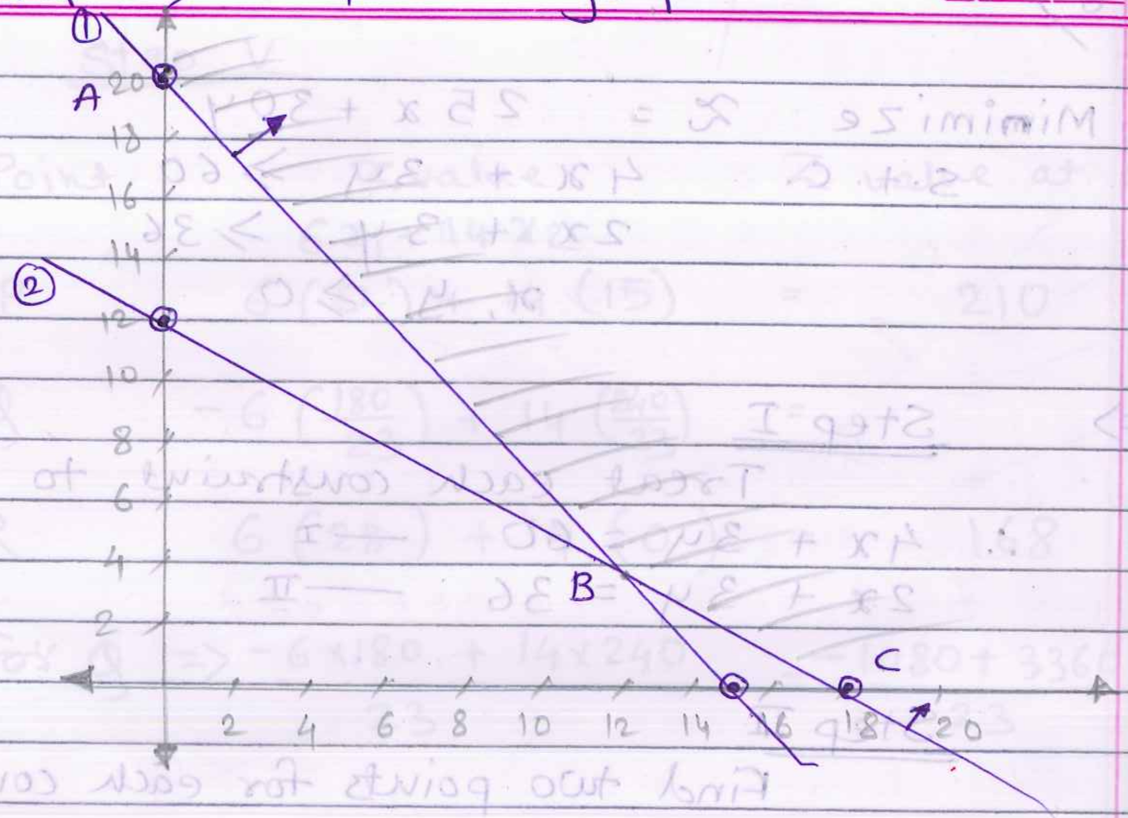
Put $y = 0$

$\therefore 2x + 0 = 36$

$x = 18$

$\therefore (18, 0)$

Step III) plot a graph.



Step IV

feasible Region is unbounded poly with following points.

A (0,20)
B (,) intersection of line 1 & 2
C (18,0)

$$B \Rightarrow \quad 4x + 3y = 60 \quad \text{—— I}$$
$$\ominus \quad \underline{{}_{-}2x + {}_{-}3y = {}_{-}36} \quad \text{—— II}$$
$$2x = 24$$
$$x = 12$$

Put $x = 12$ in eq^n. II

$\therefore \; 24 + 3y = 36$

$\therefore \; 3y = 12 \quad \therefore \; y = 4$

$\therefore$ B (12, 4)

Step V

Point	Z value $25x + 30y$	Z value at point
A (0, 20)	25 (0) + 30 (20)	600
B (12, 4)	25 (12) + 30 (4)	* 420
C (18, 0)	25 (18) + 30 (0)	450

∴ Value of Z is minimum at pt B (12, 4) which is optimum solution.

$Z_{min} = 420$ at $x = 12$ & $y = 4$

Unsolved from R.C.

Prob.7) Maximize $z = 10x_1 + 15x_2$

S.t.c. $x_1 \leq 3$

$x_2 \leq 5$

$3x_1 + 4x_2 = 29$

$x_1, x_2 \geq 0$

Sol^n = Step I

Treat each constraint as equalit

$x_1 = 3$ ——— I

$x_2 = 5$ ——— II

$3x_1 + 4x_2 = 29$ ——— III

Step II

Find two points for each constrai

→ Consider $x_1 = 3$ $\therefore$ $(3, 0)$

→ Consider $x_2 = 5$ $\therefore$ $(0, 5)$

→ Consider $3x_1 + 4x_2 = 29$

Put $x_1 = 0$

$\therefore 0 + 4x_2 = 29$

$x_2 = \frac{29}{4}$

$\therefore \left(0, \frac{29}{4}\right)$

$\downarrow$

$(0, 7.25)$

Put $x_2 = 0$

$\therefore 3x_1 + 0 = 29$

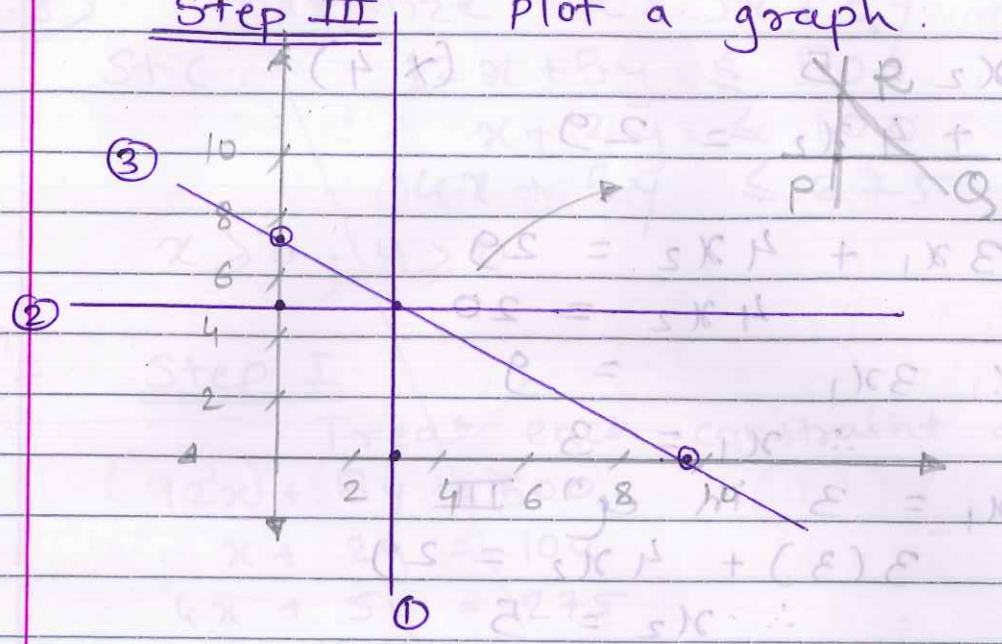
$\therefore x_1 = \frac{29}{3}$

$\left(\frac{29}{3}, 0\right)$

$\downarrow$

$(9.67, 0)$

Step III   Plot a graph.



Step IV

Draw a feasible Region.

* from graphical solution, it looks like a point only as a feasible solution

we can cross check

Intersection of

P = Line 1 & 2 = (3, 5) => by graph.

Q = Line 2 & 3 =

R = Line 3 & 1 =

Q => $x_2 = 5$ —— II × 4

$3x_1 + 4x_2 = 29$ —— III

$\therefore 4x_2 = 20$

$\underline{3x_1 \pm 4x_2 = \_29}$

$-3x_1 = -9 \quad \therefore x_1 = 3$

Put $x_1 = 3$ in eq^n III

$\therefore 3(3) + 4x_2 = 29 \quad \therefore Q = (3, 5)$

$\therefore x_2 = 5$

$R \Rightarrow \quad x_2 = 5 \qquad (\times 4)$

$3x_1 + 4x_2 = 29$

$\therefore \quad 3x_1 + 4x_2 = 29$

$\quad\quad\quad\quad\; 4x_2 = 20$

$\therefore 3x_1 \quad\quad = 9$

$\therefore x_1 = 3$

Put $x_1 = 3$ in eq^n III

$\therefore \quad 3(3) + 4x_2 = 29$

$\therefore x_2 = 5$

$\therefore R(3,5)$

$\therefore \quad P(3,5) \quad Q(3,5) \quad R(3,5)$

That is feasible ~~area~~ region is a single point with co-ordinators $(3,5)$

Step V

Point	Z value $10x_1 + 15x_2$	Z value at
$(3,5)$	$10(3) + 15(5)$	105

$\therefore \quad Z_{max} = 105$

is the optimum solution at $x_1 = 3$ & $x_2 = 5$

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Prob. 8) Maximize $Z = 3x + 5y$

St c. $12x + 5y \leq 500$

$x + 2y \leq 100$

$4x + 5y \leq 275$

$x \geq 0 ; y \geq 0$

Sol^n = Step I

Treat each constraint as equality.

$12x + 5y = 500$

$x + 2y = 100$

$4x + 5y = 275$

Step II

Find two pts. for each constraint.

→ Consider $12x + 5y = 500$

Put $x = 0$ ∴ $y = 100$

Put $y = 0$ ∴ $x = (500/12) = 41.66$.

solve same by simplex.

Unbounded sol^n.

Maximize $Z = 5x_1 + 3x_2$

s.t.c $4x_1 + 7x_2 \geq 28$

$-x_1 + x_2 \leq 5$

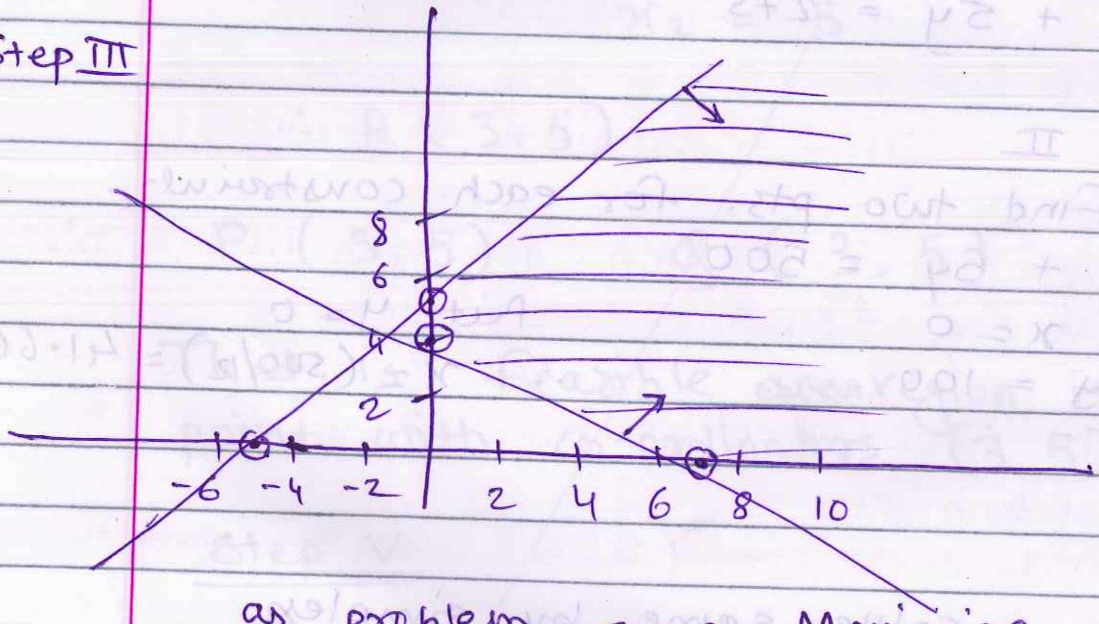
$x_1, x_2 \geq 0$

Step I $4x_1 + 7x_2 = 28$

$-x_1 + x_2 = 5$

Step II. $(0,7)$ $(4,0)$ $(0,-5)$ $(5,0)$

Step III



as problem says Maximize & feasible region is unbounded

$\therefore$ Sol^n is unbounded

Infeasible Solⁿ

Maximize $Z = 3x_1 + 4x_2$

S.t.c. $2x_1 + 3x_2 \le 36$

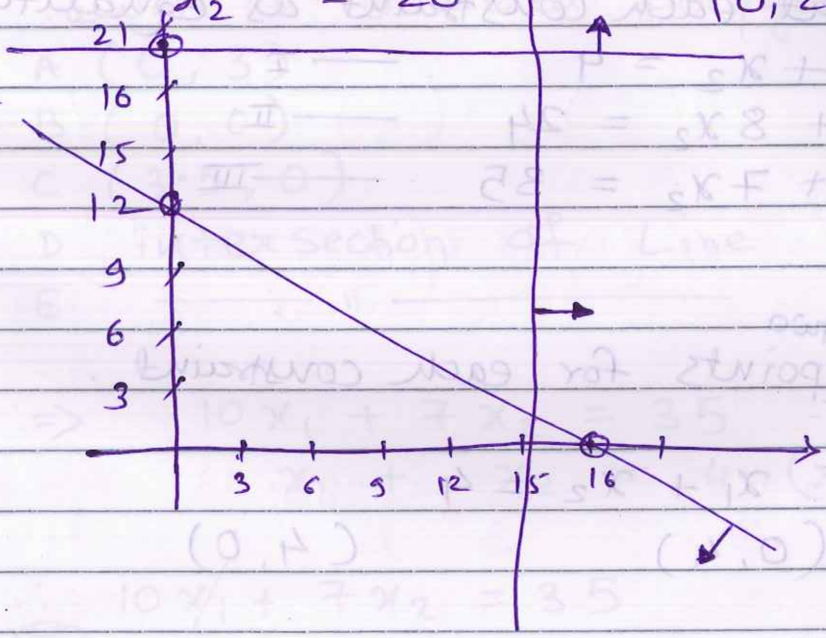
$x_1 \ge 15$

$x_2 \ge 20$

$x_1, x_2 \ge 0$

			Step II
Step I	$2x_1 + 3x_2 = 36$	(0, 18)	(12, 0)
	$x_1 = 15$	(15, 0)	// to y axis.
	$x_2 = 20$	(0, 20)	// to x axis.

Step III



Step IV We can observe that there is no common feasible region in 1st quadrant

∴ solⁿ is infeasible solution to LPP.

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Prob. 9)

Maximize $Z = 5x_1 + 7x_2$

$x_1 + x_2 \le 4$

$3x_1 + 8x_2 \le 24$

$10x_1 + 7x_2 \le 35$

$x_1, x_2 \ge 0$

Sol^n =>

Step I

Treat each constraint as equality

$\therefore \quad x_1 + x_2 = 4$ —— I

$3x_1 + 8x_2 = 24$ —— II

$10x_1 + 7x_2 = 35$ —— III

Step II

Find two points for each constraint.

→ Consider $x_1 + x_2 = 4$

$\therefore$ $(0, 4)$ $\quad$ $(4, 0)$

→ Consider $3x_1 + 8x_2 = 24$

Put $x_1 = 0$ $\quad$ Put $x_2 = 0$

$\therefore$ $8x_2 = 24$ $\quad$ $3x_1 = 24$

$x_2 = 3$ $\quad$ $x_1 = 8$

$(0, 3)$ $\quad$ $(8, 0)$

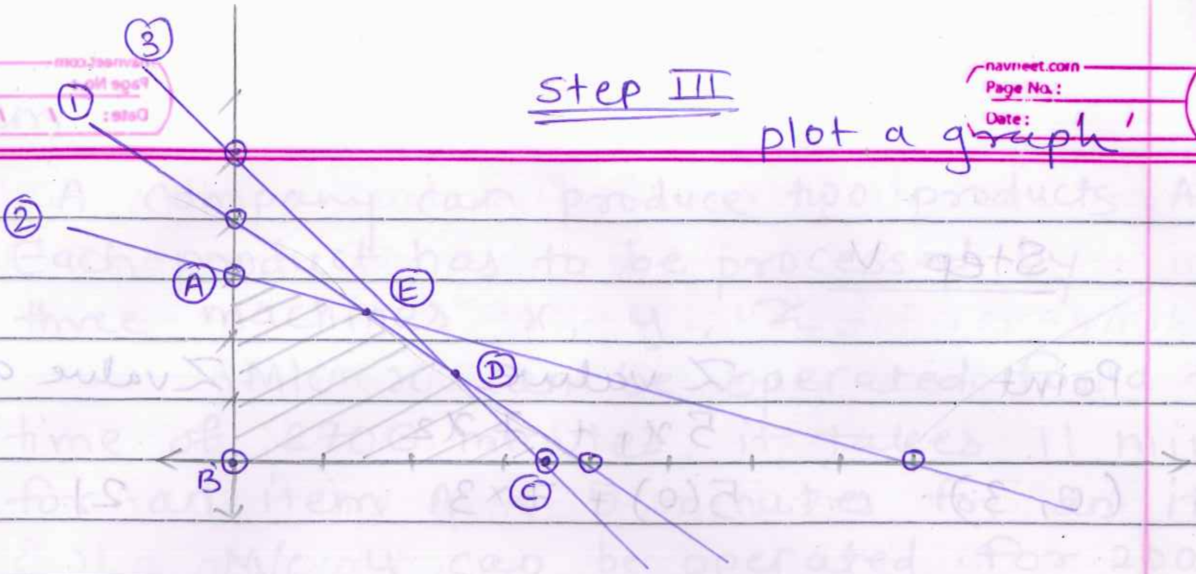
→ Consider $10x_1 + 7x_2 = 35$

Put $x_1 = 0$ $\quad$ Put $x_2 = 0$

$7x_2 = 35$ $\quad$ $\therefore$ $x_1 = 3.5$

$x_2 = 5$

$(0, 5)$ $\quad$ $(3.5, 0)$



Step III

plot a graph

Step IV

Draw a feasible region which is bounded polygon with following pts

A (0 , 3)

B (0 , 0)

C (3·5 , 0)

D intersection of Line 3 & 1

E ——— " ——— 2 & 1

for D ⇒ $10x_1 + 7x_2 = 35$

$x_1 + x_2 = 4$ (× 10) mutiply by

$\therefore$ $\cancel{10x_1} + 7x_2 = 35$

$\ominus$ $\cancel{10x_1} + 10x_2 = 40$

$-3x_2 = -5$ $\therefore x_2 = 5/3$

$\therefore x_1 = \frac{4}{1} - 5/3 = (12 - 5/3) = 7/3$

$\therefore D\left(\frac{7}{3}, \frac{5}{3}\right)$

for E ⇒ $\cancel{3x_1} + 8x_2 = 24$

$-\cancel{3x_1} + 3x_2 = 12$

$5x_2 = 12$ $\therefore x_2 = 12/5$

$\therefore x_1 = 4 - \frac{12}{5} = \frac{20-12}{5} = \frac{8}{5}$

$\therefore E\left(\frac{8}{5}, \frac{12}{5}\right)$

Step V

	Point	Z value $5x_1 + 7x_2$	Z value at
A	$(0, 3)$	$5(0) + 7 \times 3$	21
B	$(0, 0)$	$0 + 0$	0
C	$(\frac{7}{2}, 0)$	$5(\frac{7}{2}) + 0$	$35/2$ =
D	$(\frac{7}{3}, \frac{5}{3})$	$5(\frac{7}{3}) + 7(\frac{5}{3})$	$70/3$ =
E	$(\frac{8}{5}, \frac{12}{5})$	$\cancel{5}(\frac{8}{\cancel{5}}) + 7(\frac{12}{5})$	$\frac{124}{5}$

$$\frac{35 + 35}{3} = \frac{70}{3} = 23.33$$

$$\frac{40 + 84}{5} = \frac{124}{5} = 24.8$$

$\therefore Z_{max} = 24.8$
is the optimum solution at $x_1 = \frac{8}{5}$, x

Problem

A company can produce two products A & B Each product has to be processed by three machines x, y, z.

M/C x can be operated for a total time of 2700 minutes, it takes 11 minutes for an item A & 5 minutes for an item B

M/C y can be operated for 2000 mins & it takes 5 mins for item A & 10 minutes for item B.

M/C z can be operated for 450 mins & it takes 1 min for A & 2 mins for item B

The profit per item A is Rs 10 & per item B is Rs 25. Find number of units of A & B to be produced so as to maximize the profit.

Sol^n =>

M/C	Products A	Products B	M/C Available
x	11	5	2700
y	5	10	2000
z	1	2	450
Profit (Rs)	10	25	

=> let x_1 be no. of units of A to be produced & x_2 be —"— B —"—

Then we have to find x_1 & x_2 such that

Maximize, $z = 10x_1 + 25x_2$

s.t.c. $11x_1 + 5x_2 \leq 2700$

$5x_1 + 10x_2 \leq 2000$

$x_1 + 2x_2 \leq 450$

$x_1, x_2 \geq 0$

ANS. $Z_{max} = 4500$ $x = 200$ $y = 100$ Now solve →

Prob:

A manufacturer of furniture makes two chairs & tables. Processing of these products is done on two m/c A & B.

A chair requires 2 hrs on m/c A & 6 hrs on m/c B.

A table requires 5 hrs on m/c A & no time on m/c B

There are 16 hrs available on m/c A & 30 hrs on m/c B.

Profit gained by manufacturer from a chair & a table is 10 Rs & 50Rs respectively.

What should be the daily production of each of the products?

Sol^n ⇒ Lets formulate above problem

M/C	Products		Availability
	Chair	Table	
A	2	5	16
B	6	-	30
* Profit (Rs)	10	50	

let x_1 & x_2 be no. of units of chairs & tables be produced.

Hence we require to find x_1 & x_2 such

Maximize $10x_1 + 50x_2$ (objective)

s.t.c. $2x_1 + 5x_2 \le 16$

$6x_1 \le 30$

For profit Table = 50

ANS: $Z_{max} = 160$

$x_1 = 0$

$x_2 = 16/5$

Prob.

Two diff. kinds of food A & B are being considered to form a weekly diet. The minimum weekly requirements for fat, carbohydrates & protiens are 18, 24 & 16 units respectively.

One kg of food A has 4, 16, 8 units
— " — B — " — 12, 4, 16 units respectively.

The prices of food A is Rs 4 per kg
— " — B is Rs. 3 per kg

Construct the problem to minimize the cost & solve by graphical method.

Sol^n = Lets formulate as LPP

let x = kg of food A to be taken
y = — " — B — " —

	Product A	Product B	Minimum Requirement
Fats	4	12	18
Carbs.	16	4	24
Proteins.	8	16	16
Cost	4	3	

The Problem is to find x & y such that

minimize $Z = 4x + 3y$

s.t. C: $4x + 12y \geq 18$
$16x + 4y \geq 24$
$8x + 16y \geq 16$

& $Z_{min} = \frac{54}{7}$ at $x = \frac{15}{14}$ & $y = \frac{8}{7}$
$= 7.8$ 1.2 1.1

Min. $Z = 4x + 3y$

s.t.c. $4x + 12y \geq 18$

$16x + 4y \geq 24$

$8x + 16y \geq 16$

Sol^n =

Step I – $4x + 12y = 18$ —— I

$16x + 4y = 24$ —— II

$8x + 16y = 16$ —— III

Step II

I) $4x + 12y = 18$

$x = 0$ $\qquad$ $y = 0$

$\therefore 12y = 18$ $\qquad$ $4x = 18$

$y = \frac{18}{12} = \frac{3}{2}$ $\qquad$ $x = \frac{18}{4} = \frac{9}{2}$

$\therefore (0, \frac{3}{2}) \hookrightarrow (0, 1.5)$ $\qquad$ $(\frac{9}{2}, 0) \hookrightarrow (4.5,$

II) $16x + 4y = 24$

$x = 0$ $\qquad$ $y = 0$

$\therefore 4y = 24$ $\qquad$ $16x = 24$

$\therefore y = 6$ $\qquad$ $x = \frac{24}{16} = \frac{3}{2}$

$(0, 6)$ $\qquad$ $(1.5, 0)$

III) $8x + 16y = 16$

$\therefore x = 0$ $\qquad$ $y = 0$

$6y = 16$ $\qquad$ $\therefore 8x = 16$

$\boxed{\cancel{y = \frac{16}{6} = \frac{8}{3}}}$ $\qquad$ $\therefore x = 2$

$y = 1$

$x = 0$

$(0, 1)$

New

$\cancel{(8/3, 0)}$ $\qquad$ $\cancel{(0, 8/3)}$ $\qquad$ $(2, 0)$

$\hookrightarrow \cancel{(2.66, 0)} \longrightarrow \cancel{(0, 2.66)}$

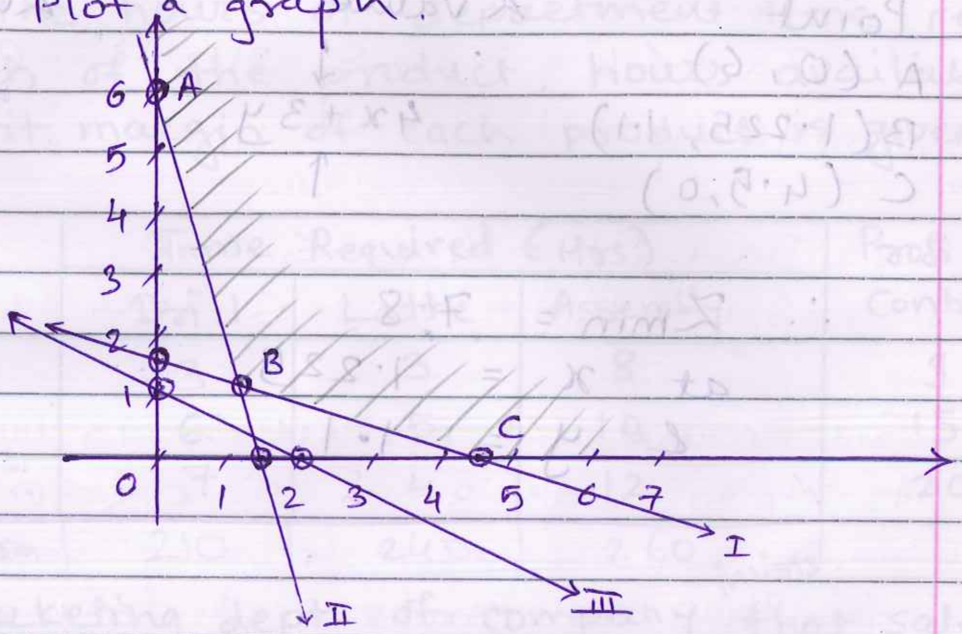
$x =$

$(2$

S

Line III) → Points (0, 1) & (2, 0)

Step III – Plot a graph.



Step IV ∴ Feasible area is unbounded polygon with vertices

A (0, 6)

B intersection of line I & II

C (4.5, 0)

For pt. B $4x + 12y = 18$

$16x + 4y = 24$

$\therefore$ $2x + 6y = 9 \quad \times 2 \Rightarrow \quad 4x + 12y = 18$

$4x + y = 6 \qquad\qquad\qquad\qquad\quad \underline{-4x + -y = -6}$

$11y = 12$

$y = 12/11 = 1.1$

Put $y = 1.1$ in eq^n II

$\therefore$ $16x + 4(1.1) = 24 \Rightarrow x = 1.225$

→

Step V

Point	Z value	Z value at
A (0, 6)		18
B (1·225, 1·1)	$4x+3y$	7·8
C (4·5, 0)		18

$\therefore \quad Z_{min} = 7{\cdot}8$

at $x = 1{\cdot}225$

& $y = 1{\cdot}1$

Prob

A company makes three products x, y, z & those goes through 3 departments drill, Lathe & Assembly.

The hours of department time required by each of the product, hours available & profit margin of each product is given below

Products	Time Required (Hrs)			Profit Contribution
	Drill	Lathe	Assembly	
x	3	3	8	9
y	6	5	10	15
z	7	4	12	20
M/C operation limitation	210	240	260	

The marketing dept. of company insists that sales potential for production x & y is unlimited but for z it is not more than 30 units. Determine the optimum production schedule.

→ Let a, b, c

Let 'a' no of units of product x to be produce
— " — b — " — product y — " —
— " — c — " — product z — " —

The Objective is to maximize total profit

∴

$$z = 9a + 15b + 20c$$

Constraints → Next page

Drill Dept -

The total time required for drilling must be less than 210 hrs.

for $3a + 6b + 7c$

$\therefore 3a + 6b + 7c \leq 210$ — ①

Simillarly

$3a + 5b + 4c \leq 240$ — ②

$8a + 10b + 12c \leq 260$ — ③

Sale count of product z is not likely to cross unit 30 $\therefore c \leq 30$

$\therefore$ Problem statement is :

Maximize $Z = 9a + 15b + 20c$

s.t.c $3a + 6b + 7c \leq 210$

$3a + 5b + 4c \leq 240$

$8a + 10b + 12c \leq 260$

$a, b, c \geq 0$

$c \leq 30$

The other fixed overhead cost for plant I & II are Rs 100 & 150 per day respectively.

Prob

A company producing three brands of shampoos has two plants located at two places. Each plant has following production capacity per day

Plants	Fresh	Bloossom	Moon
	Brands (Bottles per day)		
I	3000	1000	2000
II	1000	1000	6000

A market survey indicates in any particular three month there will be minimum demand of 24,000 bottles of Fresh, 16,000 bottles of bloossom & 48,000 bottles of Moon.

The operative cost per day of running the plants I & II are 600 monetary units & 400 monetary units respectively. How many days should the company run each plant during the month so that production cost is minimized while still meeting the market demand?

Sol^n:- Let x_1 be days for plant I

Let x_2 — " — II

$\therefore$ $600x_1 + 400x_2$ will be cost for plant I & II

also fixed overload cost is $100 + 150 = 250$ Rs

$\therefore$ Minimize $Z = 600x_1 + 400x_2 + 250$

for fresh shampoo,
total 3000 x_1 for plant I
& 1000 x_2 for plant II
& with minimum demand of 24000
∴ constraint can be expressed as

$$3000 x_1 + 1000 x_2 \geq 24000$$

i.e.

$$\cancel{1000}(3x_1 + x_2) \geq \cancel{1000}(24)$$

$$\therefore 3x_1 + x_2 \geq 24 \quad \text{—— (I)}$$

Simillarly

$$x_1 + x_2 \geq 16 \quad \text{—— (II)}$$

$$\& \; \cancel{3} \; 2x_1 + 6x_2 \geq 48\cancel{0}$$

$$x_1 + 3x_2 \geq 24 \quad \text{—— (III)}$$

∴ Problem Statement is.

Minimize $Z = 600x_1 + 400x_2 + 250$

S.t.C. $3x_1 + x_2 \geq 24$

$x_1 + x_2 \geq 16$

$x_1 + 3x_2 \geq 24$

$x_1, x_2 \geq 0$

Camlin Exam
DATE:....................

Probability

→ Uncertainty

* Trial = Prob. of experiment
* Event / Case = Outcome of trial

eg. → Throw a coin = Trial
Head / Tail = Event ; denoted by A, B, C . . . .

* Equally Likely → Head or Tail (50 - 50)

* Mutually Exclusive → throw a die, 6 faced.

means if 1 is at uppermost face ;
this will exclude occurance of 2, 3 . . . 6 .

* Exclusive → ① All possible events 1 to 6 H + T
② when 2 coins are tossed at a time
then HH, HT, TH, TT

* Sample Space → A set of all possible outcomes of a trial.
eg. Tossing a coin ⇒ Sample Space S = [H, T]
throwing a die ⇒ — " — S = [1, 2, 3, 4, 5, 6]

* Independent & Dependent Events → king 2nd successive card

* Simple & Compound Events.

single die. only '3' getting is simple

single die. getting 3 or 4' is compound event

* Probability =

$$p = P(A) = \frac{m}{n}$$

Where A = Event
p = prob.
P(A) = prob. of event A
m = no. of favorable outcome
n = Total no of possible outcomes

eg. Toss of coin.
prob of getting Head

$p = P(H) = \frac{1}{2}$ ← Possib Head
← total possible ; Head + Tail

$p = P(T) = \frac{1}{2}$

eg. Throw a die.
Prob of getting '6' at uppermost face.

$p = P(6) = \frac{1}{6}$

eg. Prob of getting Queen in well-shuffled pack

$P(A) = \frac{4}{52} = \frac{1}{13}$

A = event of getting Queen

$\bar{A}$ = non prob of A.

$$q = P(\bar{A}) = \frac{n-m}{n} = 1 - \frac{m}{n}$$

$$q = P(\bar{A}) = 1 - P(A) \quad *$$

$\therefore$ Prob of NOT getting Queen

$$= q = P(\bar{A}) = 1 - \left(\frac{1}{13}\right) = \frac{12}{13}$$

$$P(\bar{A}) + P(A) = 1$$

$$\boxed{P(A) + P(\bar{A}) = 1}$$

1)	$A \cup B$	$A + B$	A or B
2)	$A \cap B$	AB	A & B
3)	A'	$\overline{A}$	Not A
4)	$A - B$		Event A but not B

5) For mutually Exclusive events
$A \cap B = \phi$ = Null set

→ For 3 events A, B, C

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

① ADDITION THEOREM

If A & B are two events, then

$$P(A \cup B) = P(A+B)$$
$$= P(A) + P(B) - P(A \cap B)$$

eg. find prob. of getting 'king or heart card' from a well shuffled pack.

$\therefore$ King $= A$

Heart card $= B$.

$\therefore P(A \cup B) = P(\text{king}) + P(\text{Heart}) - P(\text{king \& Heart})$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$= \frac{16}{52} = \frac{4}{13}$$

eg. find prob of getting 'king or queen' card from a well shuffled pack.

king $= A$

Queen $= B$

$\therefore P(A \cup B) = P(\text{king}) + P(\text{Queen}) - P(\text{king \& Queen})$

$$= \frac{4}{52} + \frac{4}{52} - 0 = \frac{8}{52} = \frac{2}{13}$$

also as P(King & Quen) are mutually exclusive events $\therefore A \cap B = \phi$.

ex 1) In a bag containing 30 balls numbered from 1 to 30. One ball is drawn at random find prob. that no. of the ball is drawn will be

i) a multiple of 5 or 7
ii) a ——— of 4 or 6
iii) even number or multiple of 5

$\Rightarrow$ Total no of possible outcomes = $n = 30$

i) Event A = Multiples of 5

$\therefore$ set = $\{5, 10, 15, 20, 25, 30\}$ $\therefore m = 6$

$$P(A) = \frac{6}{30}$$

Event B = multiples of 7

$\therefore$ set = $\{7, 14, 21, 28\}$ $\therefore m = 4$

$\therefore P(B) = 4/30$

no commo bet^n A &

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{6}{30} + \frac{4}{30} - 0$$
$$= 10/30 = \underline{\underline{1/3}}$$

ii) A = multiple of 4

set$_A$ = {4, 8, 12, 16, 20, 24, 28} ⇒ $P(A) = 7/30$

B = multiple of 6

set B = {6, 12, 18, 24, 30} ⇒ $P(B) = 5/30$

$(A \cap B)$ = {12, 24} ⇒ $P(A \cap B) = 2/30$

$\therefore P(A \cup B) = (7 + 5 - 2)/30 = 1/3$

iii) A = even nos.

set A = {2, 4, 6, 8, 10, 12, 14 .... 30} $P(A) = 15/30$

set B = {5, 10, 15, 20, 25, 30} $P(B) = 6/30$

$A \cap B$ = {10, 20, 30} ⇒ $P(A \cap B) = 3/30$

$$\therefore P(A \cup B) = \frac{15 + 6 - 3}{30} = \frac{18}{30} = \frac{3}{5}$$

om
e
n
B.

(2) COMPOUND PROB. THEOREM

When there are two dependent events A & B prob of their simultaneous occurence is

$$P(A \cap B) = P(A) \times P\left(\frac{B}{A}\right) \text{ or } P(B) \times P\left(\frac{A}{B}\right) \longrightarrow *$$

Where $P(B/A)$ = conditional prob.

$\uparrow$ Prob of occurance of B when ~~A~~ event A already occured.

also. $P(B/A) = \frac{P(A \cap B)}{P(A)}$ —— *

→ If A & B are independent events;

$P(B/A) = P(B)$

so

$P(A \cap B) = P(A) \times P(B)$ → multiplica^n thm.

* for 3 events A, B, C

$$P(A \cap B \cap C) = P(A) \times P(B/A) \times P\left(\frac{C}{A \cap B}\right)$$

$\downarrow$ if A, B, C are independent

$$P(A \cap B \cap C) = P(A)\, P(B)\, P(C)$$

Find prob of drawing Queen & ~~king in~~ ~~that~~ order from a pack of cards in two consecutive draws when the two cards are

i) not being replaced (ii) being replaced.

(i) $P(A) = 4/52$ (event Queen A) prob.

if card not replaced, the remaining cards now are 51

& event B (draw king) is dependent on 1st

$\therefore P(B/A) = 4/51$

$\therefore P(A \cap B) = P(A) \times P(B/A) = \frac{4}{52} \times \frac{4}{51} = \frac{16}{2652}$

(ii) if card is replaced, 2nd time also 52 cards & event B nt depend on event A

$P(A) = 4/52$ $P(B) = 4/52$

as nt depend $\Rightarrow$ (multiplicⁿ thm)

$P(A \cap B) = P(A)\ P(B) = \frac{4}{52} \times \frac{4}{52} = \frac{16}{2704}$

③ ~~BAYES' THEOREM~~

ⓐ Factorial.

$n! = \lfloor n = \text{factorial } n$

$n! = n \times (n-1) \times (n-2) \times \dots 3 \times 2 \times 1$

$\therefore \quad 8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$3! = 3 \times \underline{2 \times 1}$

$\downarrow$

$3! = 3 \times 2!$

$\therefore \quad n! = n \times (n-1)!$

$0! = 1$

ⓑ Permutations →

If we have 'n' different objects & we arrange 'r' objects at a time out of n then each possible arrangement is called permutⁿ.

Notation = ${}^nP_r$

$${}^nP_r = \frac{n!}{(n-r)!}$$

$${}^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{1!} = n!$$

We can arrange letters RAM is ${}^3P_3 = 3! = 6$ different ways.

RAM RMA
ARM MRA
MAR AMR

IF $2! =$
IF
FI

eg → find $^8P_5$

$\Rightarrow {}^8P_5 = \frac{8!}{(8-5)!} = \frac{8!}{3!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3!}$

$= 6720$

eg → In how many ways we can arrange letters of the word FATHER.
In how many ways of them, letter F will be at end position?

sol^n ⇒ FATHER = 6 letters are there.
we can arrange 6 letters out of 6 in
$^6P_6 = 6!$ different ways

sol^n ⇒ If 'F' at end position = Fixed position
we have to arrange 5 letters in 5 places
in $^5P_5 = 5!$ different ways

Combination —

If out of 'n' diff objects we select 'r' objects at a time then each possible selection is called as 'combination'.

Notation = ${}^{n}C_{r}$

$$\boxed{{}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}}$$

${}^{n}C_{1} = n$

${}^{n}C_{0} = 1$

⇝ find value of ${}^{8}C_{2}$

$${}^{8}C_{2} = \frac{8!}{2!\,(8-2)!} = \frac{8!}{2!\,6!} = \frac{8 \times 7 \times 6!}{2 \times 1 \times 6!} \quad 28$$

There are 5 professors & 10 students out of whom a committee of 5 is formed.
Find in how many diff ways this can be done?

⇒ Total $15 = (10+5)$ persons.

∴ ${}^{15}C_{5}$ different ways.

$$= \frac{15!}{5!\,(15-5)!} = \frac{15!}{5!\,10!} =$$

prob.

A committee of 5 is to be formed from group of 8 boys & 7 girls.
Find prob. that committee consist of 3 boys & 2 girls.

$\Rightarrow$ 8 boys & 7 girls = 15 total.
5 is to be chossen.
In $^{15}C_5$ different ways. = Sample space

$\boxed{^{15}C_5 = S}$ —— 1

event A = committee have 3 boys & 2 girls.

$\therefore$ 3 boys out of 8 = $^8C_3$
2 girls out of 7 = $^7C_2$

$\therefore$ Event A = $^8C_3 \times {^7C_2}$ —— 2 $\Rightarrow$ multiplica^n thm.

$$\therefore P(A) = \frac{^8C_3 \times {^7C_2}}{^{15}C_5} \begin{matrix} \longrightarrow 2 \\ \longrightarrow 1 \end{matrix}$$

$$= \frac{\frac{8!}{3!(5!)} \times \frac{7!}{2!\,5!}}{\left(\frac{15!}{5!\,10!}\right)}$$

$$= \frac{8!\ \ 7!}{3!\ 2!\ 5!\ \cancel{5!}} \times \frac{\cancel{5!}\ \ 10!}{15!}$$

$$= \frac{8 \times 7 \times 6 \times \cancel{5!}}{\cancel{5!}} \ \frac{\cancel{10!}}{15 \times 14 \times 13 \times 12 \times 11 \times \cancel{10!}} \ \frac{7 \times 6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}\ \ 2!}$$

$$= \frac{8 \times 7 \times \cancel{6} \times \cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4}\ \cancel{2}}{\cancel{15} \times \cancel{14} \times 13 \times \cancel{12} \times 11 \times \cancel{2} \times 1} = \frac{8 \times 7}{13 \times 11} =$$

Wht is prob. that a leap yr selected at random will have 53 Mondays.

$\Rightarrow$ leap yr = 365 + 1 = 366 days.
weeks in yr. $52 \times 7 = 364$ days

$\therefore$ 366 - 364 = 2 days remaining.

Possible Combinations are.
sun-mon, mon-Tue, Tue-Wed, wed-Thr
Thr-Fri, Fri-Sat, Sat-Sun,
i.e. 7 = total

out of which monday present is possible 2 times.

$\therefore$ Prob that leap yr will hv 53 mondays = $\left(\frac{2}{7}\right.$

2/7

Prob. that man will be alive 25 yrs hence is 0·3 & — " — his wife — " — is 0·4

Find prob that 25 yrs hence

(i) both will be alive

(ii) only man will be alive

(iii) only woman — " —

(iv) at least one of them will be alive.

$\Rightarrow$ $P(A) = 0\cdot3$

$P(B) = 0\cdot4$

$P(\bar{A}) = P(\text{man will nt be alive}) = 1 - P(A) = 0\cdot7$

$P(\bar{B}) = P(\text{woman} \text{ — " — }) = 1 - P(B) = 0\cdot6$

(i) $P(A \cap B) = P(A) \times P(B) \longrightarrow$ multiplicaⁿ thm as independent events

$= 0\cdot3 \times 0\cdot4$

$= 0\cdot12$

(ii) only man alive means wife is not alive.

$P(A \cap \bar{B}) = P(A) \cdot P(\bar{B}) = 0\cdot3 \times 0\cdot6 = 0\cdot18$

(iii) $P(\bar{A} \cap B) = P(\bar{A}) \cdot P(B) = 0\cdot7 \times 0\cdot4 = 0\cdot28$

(iv) man alive or woman alive

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0\cdot3 + 0\cdot4 - 0\cdot12$

$= 0\cdot58$

An urn contains 8 white ~~b~~ & 3 red balls.
If two balls are drawn at random,
wht is the chance that,

i) both r white
ii) both r red
iii) ~~both~~ one of each color *
iv) both r red or both r white.

$\Rightarrow$ There are total $8+3 = 11$ balls
& only 2 balls are drawn

$\therefore {}^{11}C_2 = n = 9 = \frac{11!}{2!\,9!} = 55$

(i) both r white.
$\therefore$ 2 balls drawn from 8 white.
i.e. $8C_2$ = no of favorable outcome

$\therefore m = 8C_2 = 28$

$\therefore$ Probability $= \frac{m}{n} = \frac{28}{55}$

(ii) both r red
$\therefore$ i.e. $3C_2 = m = 3$
$\therefore$ Prob = $3/55$

(iii) 1 red & 1 white. *
$\Downarrow$ $3C_1$ $\Downarrow$ $8C_1$
as they happen simultaneously, & independent
$\therefore$ ~~Prob~~ $\therefore m = 3C_1 \times 8C_1$
$= 24$
$\therefore$ Prob = $24/55$

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(iv) both red or both white

$${}^{8}C_2 \qquad {}^{3}C_2$$

$m = 28 + 3 = 31$

$\therefore$ Prob $= \boxed{31/55}$

A card is drawn from pack of cards. What is chance of drawing a 'red queen' given that the card drawn was a 'face card'

~~Red Queen is drawn = event A~~

$\Rightarrow$ Face card is drawn = event B.

There are 'two' Red Queens $= 2$

& face cards total $= (J+Q+K) \times 4 = 3 \times 4 = 12$

$\therefore P(A) = 2/52 \qquad P(B) = 12/52$

To find, chances of occurance of event A, when event B already occured

i.e... $P(A|B) = ?$ ★ Ref. conditional Prob.

Now, $P(A \cap B) = P(B) \cdot P(A|B)$

for; $A \cap B$ = Face card & Red Queen $= 2 = m$

$\therefore P(A \cap B) = 2/52$

$$\frac{2}{52} = \frac{12}{52} \cdot P(A|B)$$

$$\therefore P(A|B) = \frac{2}{12} = \frac{1}{6}$$

A candidate is selected for interview for 3 posts. For the first post there are 3 candidates, for second there are 4 & for third there are 2..

What is prob that a candidate is selected for at least one post.

⇒ Event A = candidate is selected for 1^{st} post

B = ——— " ——— 2^{nd}

C = ——— " ——— 3^{rd}

∴ $P(A) = 1/3$ $\quad P(B) = 1/4$ $\quad P(C) = 1/2$

~~$\frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{24}$~~

P (candidate is selected for atleast one post)

= 1 − P(cand ~ not ~ any post)

$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$ as are independent events

~~$= 1 - P(\bar{A})\, P(\bar{B})\, P(\bar{C})$~~

$= 1 - \left(\frac{2}{\cancel{3}} \cdot \frac{\cancel{3}}{4} \cdot \frac{1}{\cancel{2}}\right) = \frac{3}{4}$

A movie is filled with 700 people of which 60% are females. 70% are seated into no smoking area including 300 females. A person is selected at random.

Find prob. that:

1) person is a male
2) — " — male or non-smoker
3) — " — smoker, if person is known to be male

$\Rightarrow$ Total $n = 700$

60% are female $= \frac{60}{100} \times 700 = \underline{420}$

$\therefore$ No of males $= 700 - 420 = \underline{280}$

70% are non-smokers $= \frac{70}{100} \times 700 = \underline{490}$

$\therefore$ out of 490, 300 are female.

$\therefore$ No of Non smoker females $= \underline{300}$

No of — " — Males $= 490 - 300 = \underline{190}$

$\therefore$ No of smoker male $= 280 - 190 = \underline{90}$

No of — " — Females $= 420 - 300 = \underline{120}$

$\therefore$ Total no of smokers (M+F) $= 90 + 120 = \underline{210}$

(1) P(Person is male) $= 280/700 = 4/10 = 0.4$

(2) P(Male or Non Smoker)

$= P(\text{Male}) + P(\text{non smoker}) - P(\text{Male } \& \text{ Non smoker})$

$= \frac{280}{700} + \frac{490}{700} - \frac{190}{700}$

$= \frac{580}{700} = \frac{29}{35} = 0.828$

(3) let A = person is smoker
B = — " — male.

find prob of (person is smoker, if person is male)

$$P(A/B) = ?$$

$$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B) = \frac{280}{700}, \quad P(A \cap B) = \frac{90}{700}$$

$$\therefore P(A/B) = \frac{90/700}{280/700} = \frac{90}{280} = 0.321$$

It is known that 15% of male & 10% of female in a town having equal no of them, are unemployed. A person is selected at random. What is prob that (i) He is employed, (ii) person is employed.

$\Rightarrow$ M = Male F = Female, E = employed $\bar{E}$ = unemployed

$\therefore P(M) = 1/2$, $P(F) = 1/2$ $\rightarrow$ given

also $P(\bar{E}/M) = 15\%$, $P(\bar{E}/F) = 10\%$ $\rightarrow$ given.

(i) P(~~He~~ he is employed) = P(~~person is employed given that he is ma~~

$P(E/M) = 1 - P(\bar{E}/M) = 1 - 0.15 = 0.85$

$P(E/F) = 1 - P(\bar{E}/F) = 1 - 0.1 = 0.90$.

(ii) P(Person is employed)

= P(person is male & employed) + P(person female & emplo

= $P(M \cap E) + P(F \cap E)$

= $P(M)\, P(E/M) + P(F)\, P(E/F)$

= $0.5 \times 0.85 + 0.5 \times 0.9$

= 0.875

Bayes' Thm.

If $A_1 A_2 A_3 \cdots A_k \cdots A_N$

are mutually exclusive & collectively exhaustive even[ts]
&

B is any other event, that occurs in conjunction with events $A_1 A_2 A_3 \cdots A_N$

then,

$$P(A_k/B) = \frac{P(A_k) \times P(B/A_k)}{\sum_{k=1}^{n} P(A_k) \times P(B/A_k)}$$

→ If events are 2 i.e. $A_1 A_2$

then $$P(A_1/B) = \frac{P(A_1)\ P(B/A_1)}{\sum_{k=1}^{2} P(A_k)\ P(B/A_k)}$$

$$= \frac{P(A_1) \cdot P(B/A_1)}{P(A_1) \cdot P(B/A_1) + P(A_2)\ P(B/A_2)}$$

→ If events are 3 i.e. $A_1\ A_2\ A_3$

then $$P(A_1/B) = \frac{P(A_1)\ P(B/A_1)}{P(A_1)\ P(B/A_1) + P(A_2)\ P(B/A_2) + P(A_3)\ P(B/A_3)}$$

A factory has 2 m/c A & B

A producing 300 units } Forming total o/p
B —"— 700 —"—

5%. of items produced on m/c A are defective
1%. —"— m/c B —"—

If a defective item is drawn at random, ?
1) wht is prob that it is produced by m/c A.
2) —"— M/C B.
3) —"— item drawn randomly is defecti..

⇒ let event A_1 = item produced on m/c A
A_2 = —"— B.
D = i—"— is defective

1) To find prob that item is produced on m/c A & it is defective
i.e $P(A_1/D)$ = ?

Bayes thm $P(A_1/D) = \dfrac{P(A_1)\,P(D/A_1)}{P(A_1)\,P(D/A_1) + P(A_2)\,P(D/A_2)}$

$n = 300 + 700 = 1000$ total o/p

$\therefore P(A_1) = \dfrac{300}{1000} = 0.3$, $P(A_2) = \dfrac{700}{1000} = 0.7$

also given M/c A produces 5%. defective items

$P(D/A_1) = \dfrac{5}{100} = 0.05$

$P(D/A_2) = 0.01$

$\therefore P(A_1/D) = \dfrac{0.3 \times 0.05}{(0.3 \times 0.05) + (0.7 \times 0.01)} = 0.68$

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2)

simillarly here n't asked but we can find out,

→ prob that defective item is drawn, is produced by m/c B.

i.e. $P(A_2/D) = 0.318$ H.W.

3) Prob of defective item drawn randomly from total o/p.

$P(D) = P$ (item produced on m/c A & it is defective) ★

OR P (—— " —— B —— " ——)

$\therefore P(D) = P(A_1 \cap D) + P(A_2 \cap D)$

= as these events are dependent

$= P(A_1)\,P(D/A_1) + P(A_2)\,P(D/A_2)$

$= (0.3 \times 0.05) + (0.7 \times 0.01)$

$= 0.022$

normal Simplex method

Three m/c A, B, C produce respectively 50%. 30%. & 20%. of total no of items of a factory.

The %. of defective outputs of these m/c are 3%. 4%. 5%. respectively.

If an item is selected at random. wht is prob that selected item is defective.

$\Rightarrow$ Event A = Item is produce on m/c A

B = —"— B

C = —"— C

D = —"— Defective.

Then we get

$P(A) = 0.50 \quad P(B) = 0.30 \quad P(C) = 0.20$

$P(D/A) = 3\% = 0.03$

$P(D/B) = 4\% = 0.04$

$P(D/C) = 5\% = 0.05$

$\therefore P(D) = P(D\cap A) + P(D\cap B) + P(D\cap C)$

$= P(D/A)\,P(A) + P(D/B)\,P(B) + P(D/C)\,P(C)$

$= (0.03)\cdot(0.50) + (0.04)(0.30) + (0.05)(0.2$

$P(D) = 0.037$

Prob. Distribution

1) Random Variable –

= A variable whose value is determined by the outcome of a random expt.

let $S = \{ e_1\ e_2\ e_3 \dots e_n \}$

be any sample space containing n diff. outcome

Consider real numbers $x_1, x_2 \cdots x_n$ which ar associated with these outcomes.

Then set of real no = random variable.

eg – Toss a coin $S = \{H, T\}$

If Win or Loss is assciated with outcome H, T then those values of H, T are called as Random Va

2). Discrete Variable.

= A variable which can assume only particular val

eg. no of customer arrive in 10 min

no of sim card change in 1 yr.

no of backlogs in 1 sem.

3) Continuous Variable

= A variable which can assume any value.

eg ht, wt,

Expected Value $E(X) \rightarrow$ (mean)

let X be random variable with values $x_1\ x_2 \dots x_n$
let $P(x)$ be prob. denoted by $P(x_1)\ P(x_2) \dots P(x_n)$

we arrange them as

X :	x_1	x_2	x_3	...	x_n
$P(X)$:	$P(x_1)$	$P(x_2)$	$P(x_3)$		$P(x_n)$

then Expected value of random variable x =

$$E(X) = x_1 P(x_1) + x_2 P(x_2) + \dots x_n P(x_n)$$

$$E(X) = \sum_{i=1}^{n} x_i P(x_i)$$

~~& Var(X)~~ = $\cancel{\sigma^2(X) = E(X^2) - [E(X)]^2}$

eg. find $E(X)$ for following data

X	$P(X)$	$X.P(X)$
-1	$1/3$	$-1/3$
0	$1/6$	0
1	$1/6$	$1/6$
2	$1/3$	$2/3$
		$1/2$

$\therefore E(X) = 1/2$

Prob. Distribu^n →

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= Prob Dist^n for a discrete random

Binomial Distⁿ

⇒ Consider for one expt, only 2 possible outcomes

i.e. ~~win~~ Success or ~~loss~~ Failure

If ~~then~~ prob of ~~win~~ Success p

" " ~~loss~~ Failure $= q$

then $p + q = 1$

eg. Coin toss ; $p = P(H) = 1/2$, $q = P(T) = 1/2$

* Consider such 'n' independent trials.

& let r = number of successes in 'n' trials

then prob of getting 'r' successes in 'n' trials is given by

$$P(r) = {}^{n}C_{r} \cdot p^{r} \cdot q^{n-r} \quad \text{where } r = 0, 1, 2, 3 \ldots n$$

where p = prob. of getting success in one trial

& q = " " failure "

$q = 1 - p$

Binomial Prob. Distribuⁿ is denoted as $B(n, p, r)$ — are parameters of distⁿ

* Mean $= E(r) = \bar{r} = np$
* VAR $= \sigma^2 = npq$

* No. of sets in which 'r' successes are achieved in 'n' trials, from total number of N sets is $f(r) = N.P(r)$

$$f(r) = N \cdot {}^{n}C_{r}\, p^{r} q^{n-r}, \quad r = 0, 1, 2, \ldots n$$

10 unbiased coins are tossed ~~simultaneously~~,
find prob. that there will be
(i) exactly 5 heads (ii) at least 8 heads
(iii) not more than 3 heads (iv) atleast 1 head

~~get $P(r)$ = in terms of r~~

⇒ let p = Prob of getting Head (success) in 1 trial = 1/2
q = — " — Tail (failure) — " — = 1/2

Here 10 coins are used in expt.
$\therefore n = 10$
Hence prob of getting 'r' successes (heads) in 10 trials is given by Binomial Prob. f^n as.

$$P(r) = {}^{10}C_r \, p^r . q^{10-r}$$

$$= {}^{10}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{10-r}$$

$a^m . a^n = a^{m+n}$

$$= {}^{10}C_r \left(\frac{1}{2}\right)^{r+10-r} = {}^{10}C_r \left(\frac{1}{2}\right)^{10} = \boxed{\frac{1}{1024} \, {}^{10}C_r}$$

Std. term we derive

(i) Prob. of getting exactly 5 heads
$\therefore r = 5$
$$\therefore \frac{1}{1024} {}^{10}C_5 = \frac{1}{1024} \frac{10!}{5!\,5!} = 0.246$$

(ii) Prob. of getting atleast 8 heads i.e. $P(r \geq 8)$
= P (that 'r' equal to 8 or 9 or 10)
$= P(8) + P(9) + P(10)$

$$= \frac{1}{1024}\left[{}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}\right]$$

$$= \frac{1}{1024} \times 56 = 0.055$$

iii) P(that not more than 3 heads)

i.e. $P(r \leq 3)$

$= P($ that r equal to 0 or 1 or 2 or 3 $)$

$= P(0) + P(1) + P(2) + P(3)$

$$= \frac{1}{1024}\left[\underbrace{{}^{10}C_0}_{1} + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3\right]$$

$= 0.172$

(iv) P(that at least one head)

i.e. $p(r \geq 1)$ Reverse

$= 1 - P(r < 1)$

$= 1 - P(r = 0)$

$= 1 - P(0)$

$$= 1 - \left[\frac{1}{1024}\,{}^{10}C_0\right] = 1 - \frac{1}{1024} = 0.99$$

Poisson Distⁿ

= ~~Limit edge~~ Limiting case of Binomial Distⁿ under following conditions

1) no. of trials i.e. n is very large.
2) const prob. of success for each trial i.e. 'p' is very large (near 1) or very small (near 0).
3) $np = \underset{(\text{mean})}{m} = E(r) = \bar{r}$ is finite & positive.

in short, $n \to \infty$ $p \to 0$ or 1

(i) Parameter is $m = np$

& $P(r) = \dfrac{e^{-m} m^r}{r!}$ where $r = 0, 1, 2 \ldots$

(e^{-m} values given in exam)

(ii) mean = var = $m = np$

(iii) $f(r) = N \; P(r)$

Expected freq of occurance of successes. set of trials. occurance of $\underbrace{r = \text{successes}}_{\text{random}}$.

* e = the base of natural algorithm

$e^{-5} = 0.00638 = 0.007$

$e^{-4} = 0.01832$

~~e^{3} = 0.00...~~

If 5 % of electric bulbs manufactured by a Company are defective,
~~use poisson distⁿ~~ to find,
the prob that in a box of 100 bulbs,
(i) None is defective
(ii) 3 bulbs are defective
(iii) More than 3 bulbs are defective
given $e^{-5} = 0.007$

$\Rightarrow$ p = prob that bulb is defective
$= 5\% = 5/100 = 0.05$ ($\rightarrow$ small)

n = no of bulbs in box $= 100$ ($\rightarrow$ large)

$\therefore$ It is case of poisson distⁿ
$\therefore m = np = 100 \times 0.05 = 5$

$\therefore$ Prob of getting 'r' defective bulbs in a box of ($n = 100$) bulb is

$$P(r) = \frac{e^{-m} m^r}{r!} = \frac{e^{-5} \cdot 5^r}{r!} = \underbrace{\frac{(0.007)\,5^r}{r!}}_{\text{Std. we derive.}}$$

(i) Prob that no bulb is defective

$$P(r=0) = \frac{(0.007)\,5^0}{0!} = \frac{0.007 \times 1}{1} = 0.007$$

r = occurrence of success

(ii) Prob. that 3 bulbs are defective

$$P(r=3) = \frac{(0.007)\,5^3}{3!} = \frac{0.007 \times 125}{3 \times 2 \times 1} = 0.146$$

(iii) more than 3 bulbs are defective

$$P(r>3) = 1 - P(r \leq 3)$$

$$= 1 - [P(0) + P(1) + P(2) + P(3)]$$

$$= 1 - \left[\frac{(0.007)\,5^0}{0!} + \frac{(0.007)\,5^1}{1!} + \frac{(0.007)\,5^2}{2!} + \frac{(0.007)\,5^3}{3!}\right]$$

$$= 1 - (0.007)\left[\frac{1}{1} + \frac{5}{1} + \frac{25}{2} + \frac{125}{6}\right]$$

$$= 1 - (0.007)\left[\frac{6 + 30 + 75 + 125}{6}\right]$$

$$= 0.725$$

If $(N=10)$ such boxes are delivered, which contains more than 3 defective bulbs,

$$f(r>3) = N \times P(r>3)$$

$$= 10 \times 0.725 = 7.25$$

$\Downarrow$

7 boxes contain more than 3 defective balls

An arrival of cust at petrol pump is poison distribu^n, the mean arrival rate is 5 cust every 15 mins.

find prob that there will be exactly 7 cust in 30 min period. ($\therefore e^{-10} = 4.539 \times 10^{-5}$)

$\Rightarrow$ given mean arrival rate = 5 cust every 15 min.

$\therefore$ p = prob of arrival of cust in a min = $\frac{5}{15} = \frac{1}{3}$

n = 30 mins given = total time

$\therefore$ mean = $m = np = 30 \times \frac{1}{3} = 10$

$\therefore$ 10 cust arriving in 30 min.

Now, by Poisson, Prob. that there will be exactly 7 cust in 30 mins will be

$$P(r=7) = \frac{e^{-m} m^r}{r!} = \frac{e^{-10} 10^7}{7!} = \frac{4.539 \times 10^{-5} \times 10^7}{7!}$$

$$= \frac{4.539 \times 10^2}{7!} = \frac{453.9}{7!} = 0.09$$

r = occurance of successes.

Binomial Distr^n

A particular breed of hens lay eggs four days in a week, one egg on a day.

If the poultry has 10 hens, find the prob that on a particular day poultry gets 4 eggs.

$\Rightarrow$ In a week there are 7 days & hen lays egg on 4 days.

$\therefore$ Prob that a hen lays an egg in a day = 4/7

$\therefore$ $\boxed{P = 4/7}$

$\therefore$ Prob that hen doesn't lay egg in a day =

$q = 1 - P = \boxed{3/7 = q}$

Thus P, q are not very small values & we have to find prob of 10 hens

$\therefore$ $\boxed{n = 10}$ which is nt very large value.

$\therefore$ It is prob of Binomial Distr^n

$\Rightarrow$ To find prob that poultry gets 4 eggs

i.e. $r = 4$ out of $n = 10$

$$\therefore \quad P(r=4) = {}^{n}C_{r}\, P^{r}\, q^{n-r}$$

$$= {}^{10}C_{4} \times \left(\frac{4}{7}\right)^{4} \left(\frac{3}{7}\right)^{10-4}$$

$$= \frac{10!}{4!\,6!} \times (0.5714)^{4} \times (0.4285)^{6}$$

$$= 0.137$$

extra

Normal Distribution

This is the most imp continuous prob. distⁿ
~~This is also called as Gaussian Distⁿ~~
In many cases we observe normal distⁿ.
Prob rule for normal prob distⁿ is.

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$$

where $-\infty < x < \infty$
m = mean of distribuⁿ
σ = S.D. — added page

(1)
Std. normal Deviaⁿ $z = \frac{x-m}{\sigma}$

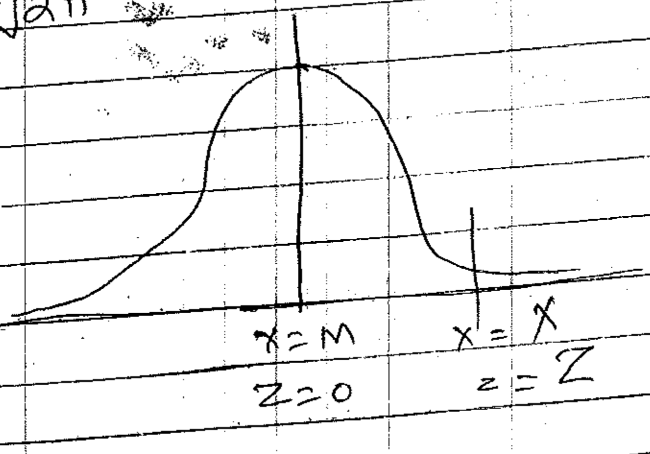
$\therefore P(x_1 \le x \le x_2) = P(z_1 \le z \le z_2)$
= Area under std. normal curve from line $z = z_1$ to $z = z_2$

Normal Prob. curve is given by eqⁿ

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}, \quad -\infty < x < +\infty$$

the std. normal prob. curve is given by

$$P(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, \quad -\infty < z < +\infty \quad \text{where } z = \frac{x-m}{\sigma}$$

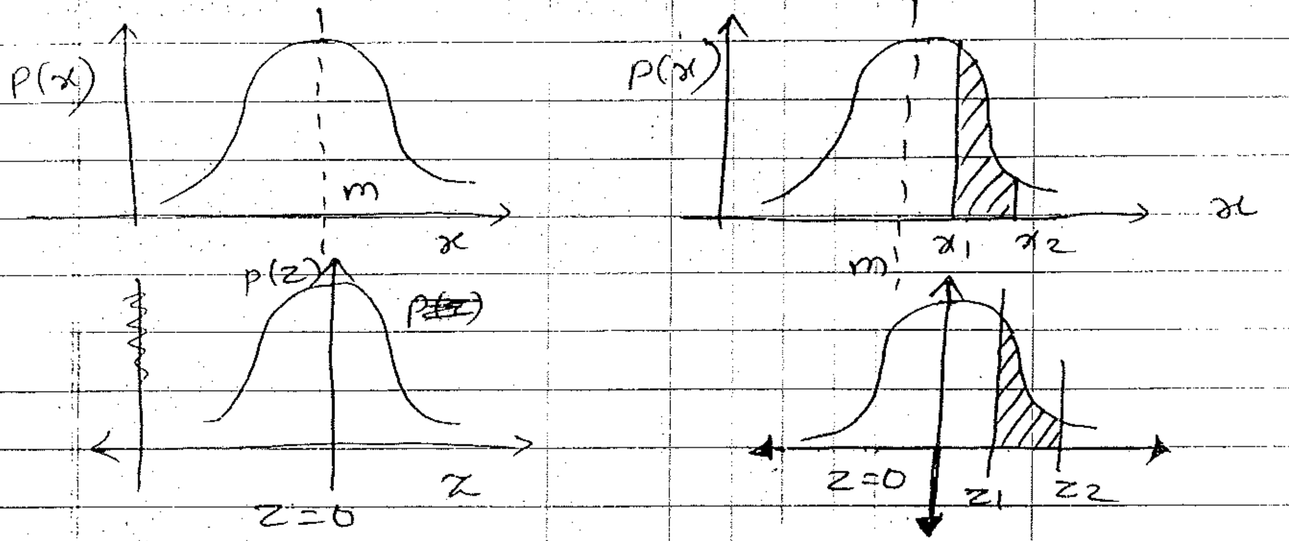


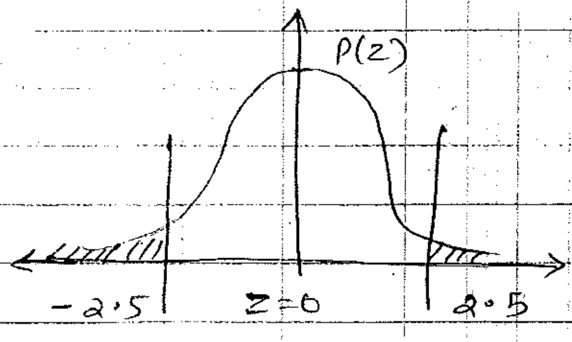
(ii) $f(x_1 \leq x \leq x_2) = NP(x_1 \leq x \leq x_2)$
$= NP(z_1 \leq z \leq z_2)$

N = total no of cases

(iii) $P(-\infty < z < \infty) = 1$
$P(-\infty < z \leq 0) = P(0 \leq z \leq \infty) = 0.5$

Normal curve is symmetric about $z = 0$.





$P(z \leq -2.5) = P(z \geq 2.5)$

The avg daily sale of 500 branch offices was Rs 150 thousand & S.D is Rs 15 thousand. Assuming Distⁿ is normal, indicate how many branches have sales between Rs 120 thousand & Rs 145 thousand.

[given

z	0.33	2.0
Area	0.1293	0.4772]

⇒ given Normal distⁿ

Let x = daily sale of a branch

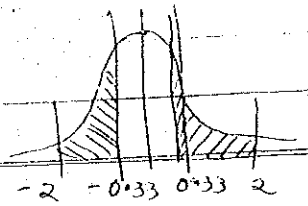
Avg daily sale $m = 150{,}000$

$\sigma = 15{,}000$

$\therefore$ Std Normal Variate $= z = \dfrac{x - m}{\sigma} = \dfrac{x - 150}{15}$

if $x_1 = 120{,}000 \Rightarrow z_1 = \dfrac{120 - 150}{15} = -2$

$x_2 = 145{,}000 \Rightarrow z_2 = \dfrac{145 - 150}{15} = \dfrac{-5}{15} = -0.33$



$\therefore$ Prob that branch has sales betⁿ $x_1 = 120$ & $x_2 = 145$

is $P(120 \le x \le 145)$

$= P(z_1 \le z \le z_2) = P(-2 \le z \le -0.33)$

$= P(0.33 \le z \le 2)$ ← by symmetry of std normal curve

easy

$= P(0 \le z \le 2) - P(0 \le z \le 0.33)$

$= 0.4722 - 0.1293$ ← given

$= 0.3479$

$\therefore$ No. of branches out of $N = 500$ having sales betⁿ Rs 120' & Rs 145000

$= N \times P(120 \le x \le 145)$

$= 500 \times 0.3479$

$= 173.95$ ⟶ Round off $=$ 174 branch

Life of car batteries is normally distributed with an avg life 5 yrs & S.D. 2 months.
What should the guarantee period be if company wishes to replace nt more than 15% of the battery. given $z = 1.04$, Area $= 0.35$

$\Rightarrow$

Let x = Life of car batteries (in months)

$\therefore$ mean of $x = m = 5$ yr $= \boxed{60}$ months

$S.D = \sigma = \boxed{2}$ months

let $z = \dfrac{x - m}{\sigma} = \dfrac{x - 60}{2}$

The given x is to be normal distribuⁿ

Now, let company guarantee period be x'

so that $z' = \dfrac{x' - 60}{2}$ —— (1)

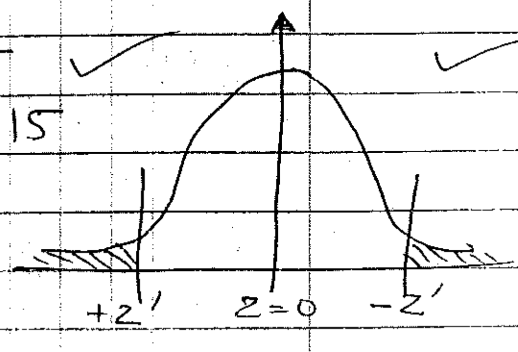
Now, company does nt want to replace more than 15% of the batteries.

Hence,
Prob that car battery will fail before this period i.e before x' should nt exceed 15% ie 0.15

means $P(x \le x') \le 0.15$

i.e. $P(z \le z') \le 0.15$

Hence
area to left to line $z = z'$ is less than area (0.5) to the left of $z = 0$.



$\therefore$ $z = z'$ must lie in left hand of normal curve

$\le$ sign $\therefore$ Left hand.
$\ge$ sign $\Rightarrow$ Rt hand.

Now $P(z \leq z') \leq 0.15$

& $\therefore$ $P(z \geq -z') \leq 0.15$ by symmetry (by graph)

$\therefore$ $0.5 - P(0 \leq z \leq z') \leq 0.15$

$\therefore$ $0.5 - 0.15 \leq P(0 \leq z \leq -z')$

$\therefore$ $P(0 \leq z \leq -z') \geq 0.35$

Ref. given means for Area 0.35, $z = 1.04$

$\therefore$ $0 \leq 1.04 \leq -z'$

$\therefore$ $-z' \geq 1.04 \Rightarrow z' \leq -1.04$ (sign change prop.) (2)

from eqⁿ (1) $\frac{x' - 60}{2} \leq -1.04$

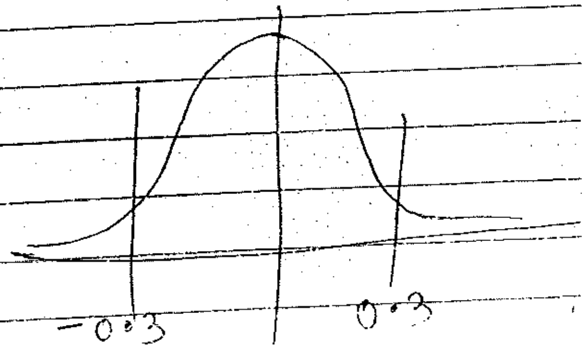
$\therefore$ $x' - 60 \leq -2.08$

$\therefore$ $x' \leq 57.92$ months

$\Downarrow$

57 months.

$\therefore$ guarantee period should be 57 months.



Of a large gr. of men 30% are under 165 cm & 60% are between 165 cm & 185 cm height. Find the mean & S.D of the gr of men assuming heights are normally distributed.

Given z : 0.525 1.28

Area under SNV from $z=0$: 0.2000 0.4000

$\Rightarrow$ Let, for given normal distn of heights (x), m = mean ht & σ = S.D of the height.

$\therefore$ The standard normal variate is $z = \frac{x-m}{\sigma}$

Now, if $x_1 = 165$ cm, $z_1 = \frac{165-m}{\sigma}$

if $x_2 = 185$ cm, $z_2 = \frac{185-m}{\sigma}$ —— (1)

Now, as 30% of men are under 165 cm,

$\therefore P(x < 165) = 30/100 = 0.3$

$\therefore P(z < z_1) = 0.3$

Thus, area to the left of $z = z_1$ is 0.3, which is less than the area of 0.5 to the left of $z = 0$

$\therefore z_1$ will lie in the left half of std normal curve

$\therefore P(z > -z_1) = 0.3$ — by symmetry

i.e. $P(-z_1 < z < \infty) = 0.3$

i.e. $P(0 < z < \infty) - P(0 < z < -z_1) = 0.3$

i.e. $0.5 - P(0 < z < -z_1) = 0.3$

$\therefore P(0 < z < -z_1) = 0.5 - 0.3 = 0.2$

$\therefore -z_1 = 0.525$ (as given $P(0 < z < 0.525)$ = Area under SNV from ($z=0$ to $z = 0.525$) = 0.2

$\therefore z_1 = -0.525$ —— (2)

Again, as 60% of men are bet^n 165 cm & 185 cm,

$P(165 < x < 185) = 60/100 = 0.6$

i.e. $P(z_1 < z < z_2) = 0.6$ using eq^n ①

$\therefore$ z_2 must lie in rt half of std normal curve,

such that, $P(z_1 < z < 0) + P(0 < z < z_2) = 0.6$

$\therefore P(0 < z < z_2) = 0.6 - P(z_1 < z < 0)$

$= 0.6 - [0.5 - P(z < z_1)]$

$= 0.6 - [0.5 - 0.3]$

$\therefore P(0 < z < z_2) = 0.6 - 0.2 = 0.4$

$\therefore z_2 = 1.28$ — ③ [as given $P(0 < z < 1.28)$ = Area under SNV from ($z=0$ to $z=0.52$) $= 0.4$]

from eq^n ① ② ③

$z_1 = \dfrac{165 - m}{\sigma} = -0.525$ & $z_2 = \dfrac{185 - m}{\sigma} = 1.28$

To get value of m, diving z_1/z_2

$\dfrac{165 - m}{185 - m} = \dfrac{-0.525}{1.28}$

$\therefore 1.28(165 - m) = -0.525(185 - m)$

$\Downarrow$

$m = 170.82$

Now if $z_2 = (185 - m)/\sigma = 1.28$

$\dfrac{185 - 170.82}{\sigma} = 1.28$

$\Downarrow$

$\sigma = 11.078$

DATE ....

(1) Bhushan Kirid. → 1) Degree Certi.
2) Org. Score CET card.
3) T.C. (S.P. clg).
4) Affdv by student
5) —— " —— parents

(2) Nikhil Nigade
1) 12th Marksheet + xerox 3.
2) 3 envelop.
3) Affd by student
4) —— " —— parent.

(3) Rohit Nanaware
1) T.C.
2) Degree (prov), 10th, 12th certificates
3)

$$200 - X$$
$$100 - ?$$
$$? = \frac{100}{200} \times X$$

11}

90/100 2/10 92/119 ob. 1 Qst.

90% 20%

SQM (Cc
A/C
BOM
LAB
OB
ECO

55

13	5	6	1	0	11
~~1~~	~~2~~	~~12~~	~~0~~	~~1~~	~~7~~
12	0	9	5	1	8
1	0	2	6	0	1
~~7~~	~~0~~	~~0~~	~~4~~	~~6~~	~~0~~
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5

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- ★ MCS
- 3 ★ SCM → Donald | closs | ropper
- Repe MTIT → N D vohra | JK sharma
- Inv. → 2 books | Jadhv
- ★ SIP

18

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Baddi
Solan
Pant Nagar
Koimbture
Bidadi / Haru
Halol
Manesar
Thirupur
H'bad (pha
Hosur (TV

(2) 15 July

15 May ★ 90% pro 16% TK
10% 90%

Simplo
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Simul.
M/C 2,3

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(3) si.
(10) Qst

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Jan

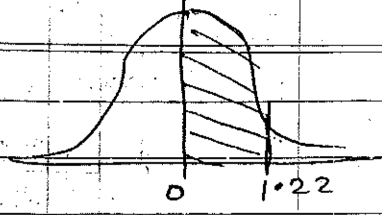
118

Write all given 1st

PAGE NO.:

DATE:

① $p(0 < z < 1.22)$

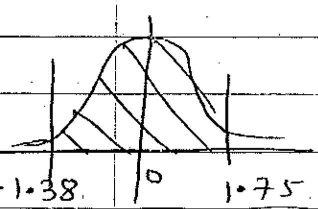


$z = 1.22$

Area $= 0.3888$

$= 0.3888$

② $p(-1.38 < z < 1.75)$



= area bet^n 0 to 1.38

+ —"— 0 to 1.75

Z	1.38	1.75
Area	0.4162	0.4599

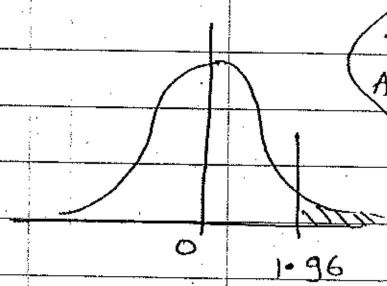
$= 0.4162 + 0.4599$

$= 0.8761$

$z = 1.96$

Area $= 0.47$

③ $p(z > 1.96)$

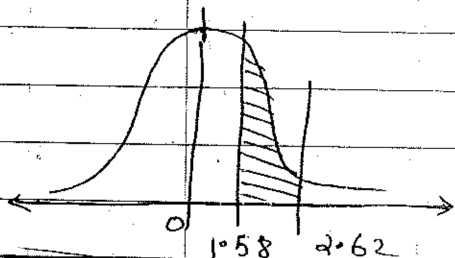


$= 0.5 -$ (Area bet^n 0 & 1.96)

$= 0.5 - 0.4750$

$= 0.0250$

④ $p(1.58 < z < 2.62)$



= Area bet^n 0 to 2.62

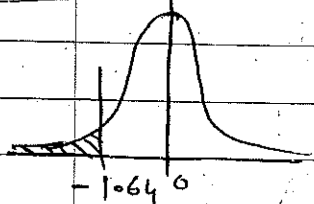
− Area bet^n 0 to 1.58

$= 0.4955 - 0.4429$

$= 0.0526$

Z =	2.62	1.58
Area	0.4955	0.4429

⑤ $P(z < 1.64)$



$= 0.5 -$ Area bet^n 0 to 1.64

$= 0.5 - 0.4495$

$= 0.0505$

$z = 1.64$

Area $= 0.4495$

prev page

★ Normal Distn.

Prop. of Normal Distn.

1) shape of curve is bell shape
2) the curve is symmetrical about 'm'
3) mean, m_d, m_o are same in normal distn
4) two ends never meet x axis
5) If sample size is large then normal distn

6) If $X \sim N(m, \sigma)$

& we define $Z = \dfrac{x - m}{\sigma}$

then $Z \sim N(0, 1)$

Z is called std. normal distribution.

* Using this property we can convert any normal distn into std normal distribu

* for std. normal variate, normally table or values are given for area under the cur when Z lies between '0' & certain point 'Z'

DTE

Queing Theory

ueue :- An arrival (units or person) has to wait if the server (service facility) is busy.
The queue is formed = waiting line.

ustomer :- Person or/units arriving at service station

ervice Station :- Point where service is provided / server

ueue length :- Number of customers waiting in queue

ueuing s/s :- entire system.
= Arrival of customer + waiting in queue + picked up for service + being served + departure of customers

(p.

IFO = First In First out
FCFS = First come First Served
LIFO = Last In First Out.

mber of service stations / servers :-
Single
Multi (series or parrallel)

rival :-
3 s. By poisson distribution.
to

Basic Notetions :-

λ = Average rate of arrival or
i.e. Avg number of customers arrive per hr / per day

μ = Average service rate or
i.e. Avg no of cust serviced per hr / per day

ρ = Traffic intensity

$$\boxed{\rho = \lambda / \mu}$$

Formulae for M/M/1/∞/FIFO
(M/M/1) : (∞/FIFO)

Where
M = Prob. distribution of no of arrivals per unit
M = — " — " — service time
* C = No. of servers
[illegible]
FIFO = service discipline

Assume :- $\lambda < \mu$

① Probability that service is busy :-
$$\rho = \frac{\lambda}{\mu}$$

② Probability that service is idle (no one in system)
$$P_0 = 1 - \frac{\lambda}{\mu}$$

③ Prob. of 'n' number of customer in system (waiting + being served)
$$P_n = P_0 \left(\frac{\lambda}{\mu}\right)^n$$

(4) Avg. waiting time of cust. in system (waiting time in queue + service time)

$$W_s = \frac{1}{\mu - \lambda}$$

(5) Avg waiting time of cust (in queue) * use if not specifi

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

(6) Avg (expected) Number of cust in system
= System length

$$L_s = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu - \lambda}$$

$$\boxed{L_s = \lambda W_s}$$

(7) Avg (expected) number of cust in Queue
= Queue Length

$$L_q = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu(\mu - \lambda)}$$

$$L_q = L_s - \frac{\lambda}{\mu} = \boxed{\lambda W_q = L_q}$$

(8) Prob. that queue size $(n) \geq k$ (Prob. that no of servers

$$P(n \geq k) = \left(\frac{\lambda}{\mu}\right)^{k+1}$$

A repairman is to be hired to repair machines which breakdown at an avg. rate of 6 per hour.
The breakdown follow poisson distribution.

Prob. 1 The unproductive time of a machine is considered

⇒ to cost Rs 50 per hour.

Two persons Mr A, Mr B are ready to take contract

Mr A charges Rs 100 per hour & services at 8 m/c per hr

Mr B — " — Rs 150 — " — — " — 12 m/c — " —

Which person should be hired?

⇒ $\lambda = 6$ per hour

Poisson Distribution = discrete

Cost per hr = 50 Rs/per = Cost of idle time of m/c

$\mu_A = 8$

$\mu_B = 12$

Cost of Repairman (A) = 100 per hr.

— " — — " — (B) = 150 per hr.

* Total Cost = cost of Repairman + cost of idle time of m/c.

for Mr. A =

Avg no of m/c in s/s (unused)

$= \frac{\lambda}{\mu - \lambda} = \frac{6}{8-6} = 3$ m/c per hr.

∴ Total m/c hour lost = 3 × 1 = 3

∴ cost per hour = 100 + (50 × 3) = 250 Rs — (1)

for Mr. B

Avg no of m/c in s/s = $\frac{\lambda}{\mu - \lambda} = \frac{6}{12-6} = 1$

∴ M/c hr. lost = 1 × 1 = 1

Cost per hr = 150 + (50 × 1) = 200 Rs — (2)

∴ As $\text{Cost}_A > \text{Cost } B$

∴ Mr B should be hired.

Prob 2

Cust. arrive at box office window being managed by single individual according to poisson process mean of 30 per hour.

The time required to serve a cust. has an exponential distance with a mean of 90 sec.

1) Find avg waiting time of cust
2) Determine avg no of cust in system
- " — avg queue length.

$\Rightarrow$ $\lambda = 30$ per hr.

$\mu =$ as given in sec., convert in hr

$$= \frac{60 \times 60}{90} = 40 \text{ per hr.}$$

1] Avg waiting time of cust.

$$W_q = \cancel{W_s - \frac{1}{\mu}} = \frac{\lambda}{\mu(\mu - \lambda)} = \boxed{0.075 \text{ hr}}$$

$$= 4.5 \text{ min}$$

2] Avg no. of cust in s/s.

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{30}{40-30} = \boxed{3} \text{ customer}$$

3] Avg queue length =

$$L_q = L_s - \frac{\lambda}{\mu} = 3 - \frac{30}{40} = \boxed{2.25}$$

Cust. arrive at service couter being served 6 1 individual at rate of 25/hr.
A server takes on an avg. 120 sec per cust.

1] Find avg waiting time of cust
2] Avg no of cust waiting in queue.

$\lambda = 25$ /hr.

$\mu = \frac{60 \times 60}{120} = 30$ /hr.

1] Avg. waiting time of cust

$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{25}{30(30-25)} = \boxed{0.16}$ hr

$= 0.16 \times 60 = 9.6$

$W_s = \frac{1}{\mu - \lambda} = \frac{1}{5} = \boxed{0.2}$ hr $= 12$ min

2] Avg no of cust in queue

$L_s = \frac{\lambda}{\mu - \lambda} = \frac{25}{30-25} = 5$

At an ATM centre arrivals occur according to poisson distribution with rate of 5 per hr. Service time per customer is distributed exponentially with mean 5 min.

1) Find avg number of cust in s/s.
2) What is % of time the facility is idle?

$\lambda = 5$ per hr.

μ = as given in ~~sec~~ min. convert in hr.

$$\frac{60}{5} = 12 \text{ cust/hr}$$

1] Avg no of cust in s/s.

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{5}{12-5} = \boxed{0.71} \text{ customers}$$

2] Avg idle time

$$P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{5}{12} = \boxed{0.59} \text{ hr}$$

& in % time = $0.59 \times 100 = 59\%$.

Prob A self service store employs one cashier at its counter. Nine customers arrive on an avg. every 5 minutes while cashier can serve 10 cust in 5 min. Assuming poisson distribution for arrival rate & exponential distribution for service rate, find

1) Avg no of cust in s/s.
2) Avg no. of cust in queue or Avg queue length
3) Avg. time a cust spends in s/s.
4) —— " —— " —— waits b4 being served

⇒ This is a case of single server Queuing Mode (M/M/1) : (∞ / FIFO)

arrival rate λ = 9 cust per unit period (5 min period).
μ = service rate = 10 cust per unit period (↓)

1) Avg no of cust in s/s = s/s length

L_s = [illegible] λ [illegible] 9 [illegible] 9

2) Avg Queue length

$$L_q = L_s - \frac{\lambda}{\mu} = 9 - \frac{9}{10} = \boxed{8.1}$$

3) Avg time cust spend in s/s.

$$W_s = \frac{1}{\mu - \lambda} = \boxed{1 \text{ unit}} \text{ of 5 mins.}$$

$$= 1(5) = \boxed{5 \text{ mins}}$$

4) Avg time cust wait b4 being served

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{9}{10(10-9)} = \frac{9}{10} \text{ units of 5 min.}$$

$$= \frac{9}{10}(5) = \boxed{4.5 \text{ mins}}$$

Multi Servers Queuing Model

$(M/M/K):(\infty/FIFO)$

~~are same~~

K = No. of Servers / (Channels) / services.

i) $\rho = \dfrac{\lambda}{K\mu}$ = Prob. that service is busy

where ρ = utilization factor.

λ = Mean arrival rate.

μ = mean service rate at each serve

ii) Prob. that there are no cust in s/s.

= s/s is idle.

$$P_0 = \left[\left\{ \sum_{i=0}^{K-1} \frac{1}{i!} \left(\frac{\lambda}{\mu}\right)^i \right\} + \frac{1}{K!}\left(\frac{\lambda}{\mu}\right)^K \frac{1}{\left(1 - \frac{\lambda}{K\mu}\right)} \right]^{-1}$$

iii) Prob that there are n units (customer) in s/s

$$P_n = P_0 \frac{(\lambda/\mu)^n}{n!} \quad \text{if } n \le K$$

$$P_n = P_0 \frac{(\lambda/\mu)^n}{K!\,(K^{n-K})} \quad \text{if } n > K$$

iv) Avg waiting time of cust in s/s.

$$W_s = \frac{\mu(\lambda/\mu)^K}{(K-1)!\,(K\mu-\lambda)^2} P_0 + \frac{1}{\mu}$$

v) Avg waiting time of cust in queue.

$$W_q = \frac{\mu(\lambda/\mu)^K}{(K-1)!\,(K\mu-\lambda)^2} P_0 = W_s - \frac{1}{\mu}$$

vi) Avg no. of cust in s/s

$$L_s = \frac{\lambda \mu (\lambda/\mu)^k}{(k-1)!\,(k\mu-\lambda)^2} P_0 + \frac{\lambda}{\mu}$$

$$L_s = \lambda W_s$$

vii) Avg no. of customers in queue

$$L_q = \frac{\lambda \mu (\lambda/\mu)^k}{(k-1)!\,(k\mu-\lambda)^2} P_0 =$$

$$L_q = L_s - \frac{\lambda}{\mu} = \lambda W_q$$

viii) Avg no. of cust in non-empty queue

$$L_w = \frac{\lambda}{k\mu - \lambda}$$

(Queue is [illegible] formed)

ix) Avg waiting time of cust (in queue) if he has to wait

$$~~W_w~~ \quad W_w = \frac{1}{k\mu - \lambda}$$

page 12.9 (R.C.) to 12.12

Decision Theory

- D.M. under Certainty
- DM under Risk ✓
- DM under un-certainty ✓
- DM under conflict

D.M. under Risk

(A) EMV criterion

Expected Moneytory Value = EMV

eg. for following profit table find EMV & optimum strategy

	States		
	N_1	N_2	N_3
	Prob. of states		
Strategies	0·3	0·6	0·1
S_1	20	18	−9
S_2	25	15	10
S_3	40	−10	12

Step I ⇒ $EMV(S_1) = 0{\cdot}3(20) + 0{\cdot}6(18) + 0{\cdot}1(-9)$

$= \boxed{15{\cdot}9}$

$EMV(S_2) = 0{\cdot}3(25) + 0{\cdot}6(15) + 0{\cdot}1(10) = \boxed{17{\cdot}5}$

$EMV(S_3) = 0{\cdot}3(40) + 0{\cdot}6(-10) + 0{\cdot}1(12) = \boxed{7{\cdot}2}$

Step II ⇒ Identify Maximum EMV

$EMV(S_2) = 17{\cdot}5$

Step III ⇒ Largest EMV corrosponds to optimal str.

∴ S_2 is the optimal Stategy.

(B) Expected value with perfect information.

⇒ for state N_1
maximum possible payoff is among
(0.3×20) (0.3×25) (0.3×40)
i.e. $(0.3 \times 40) = 12$
i.e. for strategy S_3

Similarly for state N_2
max. possible payoff is strategy $S_1 = (0.6 \times 18)$

for N_3 ; ⇒ strategy $S_3 = (12 \times 0.1)$

∴ Expected value with perfect info
$= (0.3 \times 20) + (0.6 \times 18) + (0.1 \times 12) = \boxed{24}$

(c) Value of Perfect Information.

∴ VPI = (Expected Value with perfect info)
− (Max EMV)

$VPI = 24 - 17.5 = \boxed{6.5}$

A toy company bringing a new type of doll. The company is attempting to decide whether to bring out a full, Partial or minimal product line.

The company has three levels of product acceptance & has estimated their probabilites given below.

Management will make its decision on basis of maximising the expected profit.

For the data shown below.

1) Find which product line is to be selected using EMV

2) Find value of perfect Info.

Product Acceptance	Prob.	Product Line: Full	Partial	Minimal
Good	0·2	80	70	50
Fair	0·4	50	45	30
Poor	0·4	-25	-10	0

⇒ 1) EMV (Full) = 0·2(80) + 0·4(50) + 0·4(-25) = 26

EMV (Partial) = 0·2(70) + 0·4(45) + 0·4(-10) = $\boxed{28}$

EMV (Minimal) = 0·2(50) + 0·4(30) + 0·4(0) = 22

∴ Select 'Partial' Product line as EMV(Partial) = 28.

2) VPI = Expected value with perfect Info − Max EMV

= 36 − 28 = $\boxed{8}$

as

∴ Expected value with perfect Info =

~~(0·4×50) + (0·4×45)~~ +

For good ⇒ $\boxed{0.2 \times 80}$ Fair ⇒ $\boxed{0.4 \times 50}$ Poor $\boxed{0.4 \times 0}$

16 + 20 + 0 = 36

D.M under Uncertainty :-

A] Maximin (for gain) or Minimax (for loss) Criterion

B] Maximax (for gain) or Minimin (for loss) Criterion

C] Hurwicz Alpha Criterion

D] Laplace Criterion

E] Minimax Regret Criterion

A] Maximin (for profit / gain) :- for Profit matrix

Strategies	N_1	N_2	N_3	N_4	min profit	maximin
S_1	30	10	10	8	8 *	
S_2	40	-15	5	7	-15	$\boxed{8}$
S_3	50	20	-6	10	-6	

∴ Adopt optimal strategy $\boxed{S_1}$

A] Minimax (for loss) :- for loss matrix

Sts.	N_1	N_2	N_3	N_4	max loss.	minimax
S_1	1	2	9	7	9	
S_2	8	-3	2	1	8	$\boxed{5}$
S_3	4	2	5	-3	5 *	
S_4	-3	0	6	1	6	

∴ Adopt optimal Strategy $\boxed{S_3}$

B] Maximax (for profit (gain))

Str.	States N_1	N_2	N_3	maximum profit	maximax
S_1	30	40	50	50	$\boxed{90}$
S_2	60	40	80	80	
S_3	70	90	40	90 *	

$\therefore$ Optimal Str = $\boxed{S_3}$

B] Minimin (for loss)

Str.	States N_1	N_2	N_3	min. loss	minimin
S_1	1	5	7	1	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	

$\therefore$ Optimal Str. $\boxed{S_1}$

c] Hurwicz Alpha Criterion.

= Realistic Approach

Avoid pessimistic (maximin) & optimistic (maximax)
निराशावादी आशावादी

Let α = Coefficient of optimism

* Expected Value = $\boxed{\alpha M + (1-\alpha) m}$

where M = Maximum pay~~off~~

m = minimum payoff

	N_1	N_2	N_3	N_4	Max (M)	Min (m)
S_1	30	10	10	8	30	8
S_2	40	-15	5	7	40	-15
S_3	50	20	-6	10	50	-6

given $\boxed{\alpha = 0.7}$ & $1-\alpha = 0.3$

∴ Expected Profit ~~sar~~ for S_1 = $0.7(30) + 0.3(8) = 23.4$

—— " —— S_2 = $0.7(40) + 0.3(-15) = 23.5$

—— " —— S_3 = $0.7(50) + 0.3(-6) = \boxed{33.2}$

∴ The strategy corresponding to the maximum expected profit of 33.2 ie

$\boxed{S_3 \text{ is Optimal Str.}}$

Using Hurwicz ; $\alpha = 0.6$

States	Actions A_1	A_2	A_3	A_4	Max (M)	Min (m)
S_1	10	5	8	6	10	5
S_2	3	9	15	2	15	2
S_3	-3	4	6	10	10	-3

$\alpha = 0.6 \quad \therefore \quad 1-\alpha = 0.4$

$\therefore$ Expected Profit $(S_1) = 0.6(10) + 0.4(5) = 8$

$(S_2) = 0.6(15) + 0.4(2) = 9.8$

$(S_3) = 0.6(10) + 0.4(-3) = 4.8$

$\therefore$ $\boxed{S_2 \text{ is Optimal Strategy}}$

D] Laplace Criterion.

* Avg expected payoff = $\frac{\text{Sum of all pay off}}{\text{Total no of States}}$

Stra.	States N_1	N_2	N_3	N_4	Avg. Expected payoff
S_1	15	12	18	40	$\frac{(15+12+18+40)}{4} = 21.25$
S_2	5	−2	25	12	$\frac{(5-2+25+12)}{4} = 10$
S_3	13	17	12	20	$\frac{(13+17+12+20)}{4} = 15.5$

Str S_1 gives corrosponding maximum avg expected profit = 21.25

$\boxed{\text{Optimal str} = S_1}$

E) Minimax Regret Criterion

Regret = Opportunity loss

	N_1	N_2	N_3	N_4
S_1	30	10	10 ✓	8
S_2	40	-15	5	7
S_3	50 ✓	20 ✓	-6	10 ✓

Consider Column* ⇓ convert profit (payoff) into opportunity loss

	N_1	N_2	N_3	N_4
S_1	50-30	20-10	10-10	10-8
S_2	50-40	20-(-15)	10-5	10-7
S_3	50-50	20-20	10-(-6)	10-10

⇓

	N_1	N_2	N_3	N_4	max	min
S_1	20	10	0	2	20	$\boxed{16}$
S_2	10	35	5	3	35	
S_3	0	0	16	0	16	

∴ S_3 is optimal strategy.

A farmer wants to decide which of 3 crops he should plant. The farmer has categorized the amount of rainfall is high, medium, low. Estimated profit is given below.

Estimated Profit (Rs)

Rainfall	Crop A	Crop B	Crop C
High	8,000	3500	5000
Medium	4500	4500	4900
Low	2000	5000	4000

Farmer wishes to plan one crop, decide using

1) Hurwicz Criteria (take degree 0.6)
2) Laplace criteria
3) Minimax regret criteria.

problem = ₹ '00

Str.	High	Med.	Low	M	m
Crop A	80	~~3~~45	20	80	20
Crop B	35	45	50	50	35
Crop C	50	49	40	50	40

(1) Expected Value $= \alpha M + (1-\alpha) m$

$\alpha = 0.6 \quad \therefore \quad 1-\alpha = 0.4$

Expected Profit for Crop A =

$0.6(80) + 0.4(20) = 56$

" Crop B = $0.6(50) + 0.4(35) = 44$

Crop C = $0.6(50) + 0.4(40) = 46$

∴ farmer wish to plan Crop A because of high profit expected ~~from Crop A~~ i.e. 5600

Wrong

(2) L criterio. ₹'00

	High	Med	Low	Avg expected payoff
A	80	~~35~~ 45	~~50~~ 20	$(80+\cancel{35}+\cancel{50})/3$ (45+20) = ~~55~~ 45.34
B	~~45~~ 35	~~45~~ 45	~~49~~ 50	$(45+\cancel{45+49})/3$ (35+50) = ~~46.34~~ 43.34
C	~~20~~ 50	~~50~~ 49	~~40~~ 40	$(\cancel{20}+50+49)/3$ (40) = ~~36.67~~ 46.34

∴ Str. of Crop A should be plan as expected profit = 5500 4534

Wrong

(3). Minimax Regret Criterion.

	High	Med	low
A	80	~~35~~	~~50~~ 20
B	~~45~~	~~45~~	~~49~~
C	~~20~~	~~50~~	~~40~~

Convert profit matrix into opp. loss matrix

		High	Med.	low	max.	minimax
A	(80-80)	0	15	0	~~15~~ 30	
B	(80-45)	~~35~~ 45	~~0~~ 5	~~1~~ 0	~~35~~ 45	15
C	(80-~~20~~)	~~60~~ 30	0	~~10~~ 10	~~60~~ 30	

∴ Crop ~~A~~ must be selected.
Crop C

66

In toy manufacturing company, suppose the product acceptance prob are not known, but from data given below,

~~Calculate~~
Determine optimal str. under following criterion
(i) maximax (ii) maximin (iii) minimax regret

Product	Acceptance		
	Full	Partial	Minimal
Good	8	70	50
Fair	50	45	40
Poor	−25	−10	0

(i) & (ii)

			max	min
8	70	50	70	8
50	45	40	50	40
−25	−10	0	0	−25

maximax = 70
~~minimax~~
maximin = ~~−25~~ 40

∴ By Maximax, 'Good' product str. to be selected
By maximin 'Fair' ——— " ———

(iii) Regret.

	Full	Partial	Minimal	max	minimax
Good	42	0	0	42	
Fair	0	25	10	25	25
Poor	75	80	50	80	

∴ By minimax Regret, 'Fair' product str. to be Selected

Problem statement (13.9)

prob. of demand	0.2	0.4	0.1	0.3
Demand for car each day	1	2	3	4

N_1 = State that demand is 1 ...
S_1 = Str. of buying 1 car ...
N_2 = State that demand is 2
S_2 = Str. of buying 2 cars

Payoff Profit
= (car sold) (profit per car) − (car unsold) cost
= car sold (4 L) − car unsold (6 L)

$\boxed{4 \text{ (car sold)} - 6 \text{ (car unsold)}}$

	(0.2) N_1	(0.4) N_2	(0.1) N_3	(0.3) N_4 ← Prob
S_1	× →			
S_2		× →		
S_3			× →	
S_4				×

$S_1 N_1 \Rightarrow$ 1 (4) − 6 (0) = 4
$S_1 N_2$ = 1 (4) − 6 (0) = 4
$S_1 N_3$ = 4
$S_1 N_4$ = 4

continue

$S_2 N_1$ = 1 (4) − 6 (2 − 1) = −2
$S_2 N_2$ = 2 (4) − 6 (0) = 8
$S_2 N_3$ = 2 (4) − 6 (0) = 8
$S_2 N_4$ = 8

continue

$S_3 N_1 = 1(4) - 6(3-1) = -8$
$S_3 N_2 = 2(4) - 6(3-2) = 2$
$S_3 N_3 = 3(4) - 6(3-3) = 12$
$S_3 N_4 =$ 12 continue.

$S_4 N_1 = 1(4) - 6(4-1)$ -14
$S_4 N_2 = 2(4) - 6(4-2)$ -4
$S_4 N_3 = 3(4) - 6(4-3)$ 6
$S_4 N_4 = 4(4) - 6(4-0)$ 16

	(0·2) N_1	(0·4) N_2	(0·1) N_3	(0·3) N_4
S_1	4	4	4	4
S_2	−2	8	8	8
S_3	−8	2	12	12
S_4	−14	−4	6	16

$EMV(S_1) = 4(0.2) + 4(0.4) + 4(0.1) + 4(0.3) =$
$(S_2) = -2(0.2) + 8(0.4) + 12(0.1) + 8(0.3) = 6$
$(S_3) = -8(0.2) + 2(0.4) + 12(0.1) + 12(0.3) = 4$
$(S_4) = -14(0.2) + -4(0.4) + 6(0.1) + 16(0.3) =$

$\therefore$ S_2 is optimal str. $\Rightarrow$ 6 L

Expected Moneytary Value with perfect Info.
$= 0.2(4) + 0.4(8) + 0.1(12) + 0.3(16) = 10$ L.

$\therefore$ Value of Perfect Info = $10 - 6 = \underline{4\ L}$

$\therefore$ fees are $1L < \underline{4L}$ $\therefore$ Suresh should pay f for perfect Info.

The past exp. shows that no of copies of tax laws in demand vary betⁿ 25 & 30 copies. Some taxes change every yr as such if the copies are not sold during the yr its value is reduced.

Some agency purchases such unsold copies for Rs 30. The vendor purchases the copies at Rs 80 each & sell them at Rs 100 each (PV)

Prepare pay off table for his purchasing copies betⁿ 25 & 30 & state different decision that will be taken under

① maximax ② Maximin ③ L
④ Regret if prob of demand is as follows :
⑤ EMV

Demand	25	26	27	28	29	30
Prob	0.05	0.10	0.35	0.30	0.15	0.05

⇒ let N_1 = State of Nature that demand 25 copies.
N_2 = 26
N_3 = 27
N_4 = —— " —— 28 —"—
N_5 = 29
N_6 = 30

let S_1 = Str. of purchasing 25 copies
S_2 26
S_3 27
S_4 —— " —— 28 —"—
S_5 29
S_6 30

/0

Profit per copy sold = 100 − 80 = 20/

Loss per copy sold = 80 − 30 = 50/

∴ Pay off profit = (Copies sold) (profit per copy) − (copies unsold) (loss per

∴

		N_1	N_2	N_3	N_4	N_5	N_6
25	S_1	500	500	500	500	500	500
26	S_2	450	520	→			
	S_3	400	470	540	→		
	S_4	350	420	490	560	→	
	S_5	300	370	440	510	580	580
	S_6	250	320	390	460	530	600

$S_1N_1 \Rightarrow 25(20) - \underbrace{0}_{(25-25)}(50) = 500$

$S_2N_1 \Rightarrow 25(20) - (26-25)(50) = 450$

$S_2N_2 \Rightarrow 26(20) - (\cancel{26-25})(50) = 520$ (26−26) 0

$S_3N_1 \Rightarrow 25(20) - (27-25)(50) = 400$

$S_3N_2 \Rightarrow 26(20) - (27-26)(50) = 470$

$S_3N_3 \Rightarrow 27(20) - (27-27)(50) = 540$ 0

$S_4N_1 \Rightarrow 25(20) - (28-25)(50) = 350$

$S_4N_2 \Rightarrow 26(20) - (28-26)(50) = 420$

$S_4N_3 \Rightarrow 27(20) - (28-27)(50) = 490$

$S_4N_4 \Rightarrow 28(20) - (28-20)(50) = 560$

↓

and so on →

① maximax ⇒ 600

maximum profit 600 for str. S_6

∴ Vendor should purchase <u>30 copies</u>.

② Maximin ⇒ 500, 520, 540, 560, 580, 600 Max

500 maximin ⇒ S_1

∴ Vendor should purchase <u>25 copies</u>.

③ $\mathcal{L}$ avg are

S_1 = 500

S_2 = 516.5

S_3 = 526

S_4 = 511

S_5 = 475

S_6 = 428.5

∴ Vendr go for str S_3 i.e. purchase <u>27 copies</u>

④ Regret matrix

	N_1	N_2	N_3	N_4	N_5	N_6
S_1	0	0	0	0	0	0
S_2	50	0	0			
S_3	100			0		
S_4	150				0	
S_5	200					0
S_6	250					

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Camlin Exam

DATE: ....

(4) Regret Matrix (* consider Column)

	N_1	N_2	N_3	N_4	N_5	N_6	Max
S_1	0	20	40	60	80	100	10
S_2	50	0	20	40	60	80	8
S_3	100	50	0	20	40	60	10
S_4	150	100	50	0	20	40	150
S_5	200	150	100	50	0	20	200
S_6	250	200	150	100	50	0	25

minimax = 80

$\therefore$ S_2 Str.

Vendor should purchase 26 copies.

(5) EMV

$EMV(S_1) = 0.01(500) + 0.1(500) + 0.35(500)$
$\quad + 0.3(500) + 0.15(500) + 0.05(500)$
$\quad = 500$

$EMV(S_2) = 516.5$
$(S_3) = 526 \Rightarrow$ Str. $S_3 \Rightarrow$ Should Purchase 27 copies
$(S_4) = 511$
$(S_5) = 475$
$(S_6) = 428.5$

Spade - ♤

~~Diamond~~ ♢

Heart ♡

Club ♧

Ace, King, Queen, Jack

BQ have -ve opportunity cost.

Draw polygon.

* Start in BQ (as -ve value)
* Ending & starting pt must be same.

* U can pass thr any cell
* ~~Do~~ Can't take turn 90° from "Non occupied cell"
* Only Horizontal or vertical ~~sides~~ lines drawn.
* Achieve closed polygon.

	P	Q	R	S
A	•	15	45	•
B	− 40	+ •	•	•
C	•	30	•	•
D	+ 10	− 10	•	50

This is the only option; we can draw.

* Always all other polygon corner have occupied cell.
* Start with (+) sign & follow (−) alternate
  Assign sign only at corners.

(A) Identify −ve sign with Min Assignment.
Add those to +ve sign corner
subtract ~~—~~ −ve sign corner.

	P	Q	R	S		
A	[10]	(15) [14]	(45) [7]	[15]	~~60~~ 60	$u_1 = 14$
B	(30) [7]	(10) [11]	[13]	[17]	~~40~~ 40	$u_2 = 11$
C	[12]	(30) [8]	[16]	[10]	30	$u_3 = 8$
D	(20) [8]	[15]	[11]	(50) [12]	70	$u_4 = 12$
DEM	50	55	45	50	200	

$v_1 = -4$ $v_2 = 0$ $v_3 = -7$ $v_4 = 0$

There is no degeneracy
as $7 = (4+4-1) = 7$.

AP $= 10 - (-4+14) = +0$
AS $= 15 - (14+0) = +1$
BR $= 13 - (11-7) = +9$
BS $= 17 - (11+0) = +6$
CP $= 12 - (8-4) = +8$
CR $= 16 - (8-7) = +15$
CS $= 10 - (8+0) = +2$
DQ $= 15 - (12+0) = +3$
DR $= 11 - (12-7) = +6$

All opp. cost are +ve
∴ sol^n is optimum

∴ Opt. $Xport^n$ cost
$= (30\times7) + (20\times8)$
$+ (15\times14) + (10\times11)$
$+ (30\times8) + (45\times7)$
$+ (50\times12)$
$= 1845/-$

Final ANS

ob 11

ants	P	Q	R	S	SUP	
A	40 $\boxed{48}$	20 $\boxed{51}$	× $\boxed{57}$	× $\boxed{57}$	60	3,6,10↑,–
B	× $\boxed{59}$	10 $\boxed{57}$	75 $\boxed{50}$	5 $\boxed{54}$	90	4,4,3,3
C	× $\boxed{57}$	× $\boxed{63}$	× $\boxed{65}$	50 $\boxed{59}$	50	2,4,4,4
m	40	30	75	55	200	
	9↑,–,–,–	6,6,6,6↑	7,7↑,–,–	2,2,2,5↑		

$\therefore TTC = (40 \times 48) + (20 \times 5) + (10 \times 57) + (75 \times 50) + (5 \times 54) + (50 \times 59)$

$= 10480/-$

There is No

ep II) : $6 \overset{?}{=} (4+3-1) = 6$ yes $\therefore$ Degeneracy

ep III)

	P	Q	R	S	
A	$\boxed{48}$ ✓	$\boxed{51}$ ✓	$\boxed{57}$	$\boxed{61}$	$u_1 = -6$
B	$\boxed{59}$	$\boxed{57}$ ✓	$\boxed{50}$ ✓	$\boxed{54}$ ✓	$u_2 = 0$
C	$\boxed{57}$	$\boxed{63}$	$\boxed{65}$	$\boxed{59}$ ✓	$u_3 = 5$

$V_1 = 54$ $V_2 = 57$ $V_3 = 50$ $V_4 = 54$

$48 = V_1 - 6$

$\therefore V_1 = 48 + 6 = 54$

$\therefore$ AR $= 57 - (50 - 6) = +13$

AS $= 61 - (54 - 6) = +13$

BP $= 59 - (54 + 0) = +5$

CP $= 57 - (54 + 5) = -2$

CQ $= 63 - (57 + 5) = +1$

CR $= 65 ~~50~~ - (50 + 5) = +10$

As there is -ve value $\therefore$ sol^n^ not optimum

	P	Q	R	S		
A	− (30) 48	(30) + 51	57	61	60	$u_1 = 0$
B	59	− 57	(75) 50	(15) + 54	90	$u_2 = 4$
C	+ (10) 57	63	65	(40) − 59	50	$u_3 = 9$
	40	30	75	55	200	
	$v_1 = 48$	$v_2 = 51$	$v_3 = 46$	$v_4 = 50$		

$\therefore$ AR $= 57 - (46 + 0) = +11$

AS $= 61 - (50 + 0) = +11$

BP $= 59 - (48 + 4) = +7$

BQ $= 57 - (51 + 4) = +2$

CQ $= 63 - (51 + 9) = +3$

CR $= 65 - (46 + 9) = +10$

$\therefore$ sol^n^ is optimum

Optimum TTC $= (30 \times 48) + (30 \times 51) + (75 \times 50) + (15 + 54) + (40 \times 59) + (10 \times 57)$

$= 10460/-$

Prob 12

Step I VAM

Plants	A	B	C	D	SUP	
P	30	37	38	31 (200)	200	1, -, -, -
Q	33 (250)	40	35	39 (50)	300	2, 2, 6, 1
R	39	42 (350)	32 (100)	45 (50)	500	7, 7, 3, 3
DEM	250	350	100	300	1000	
	3, 6, 6, -	3, 2, 2, 2	3, 3, -, -	8, 6, 6, 6		

TTC = (250 × 33) + (350 × 42) + (100 × 32) + (200 × 31) + (50 × 39) + (50 × 45) = 36550

Step II 6 = (3 + 4) − 1 = 6 yes

∴ there is no Degeneracy

Step III Optimality test

	A	B	C	D	
P				✓ 31	$u_1 = 31$
Q	✓ 33			✓ 39	$u_2 = 39$
R		✓ 42	✓ 32	✓ 45	$u_3 = 45$

$V_1 = -6$ $V_2 = -3$ $V_3 = -13$ $V_4 = 0$

$PA = 30 - (31-6) = +5$

$PB = 37 - (31-3) = +9$

$PC = 38 - (31-13) = +20$

$QB = 40 - (39-3) = +4$

$QC = 35 - (39-13) = +9$

$RA = 39 - (45-6) = +0$

As there is no –ve value ∴ sol^n optimum.

∴ Optimum Xport^n cost = 36550/-

Prob 13 When SUP ≠ DEM

if SUP = 12 + 28 + 30 = 70
& DEM = 25 + 35 + 30 = 90

~~There is short '20'~~
∴ ~~add 20~~

The total DEM exceeds total SUP by 20 goods. Hence 1 Dummy row of '20' supply capacity is added; for which unit Xport^n cost = '0'

Step I VAM

Plants	A	B	C	SUP	
P	12 [60]	• [72]	• [63]	12	3, 3, –, –
Q	13 [68]	15 [65]	• [69]	28	3, 3, 3, 4
R	• [77]	• [70]	30 [68]	30	2, 2, 2, 2
Dummy	• [0]	20 [0]	• [0]	20	0, –, –, –
DEM	25	35	30	90	
	60, 8, 9	65, 5, 5	63, 5, 1		
	–	5	1		

$$TTC = (12 \times 60) + (13 \times 68) + (15 \times 65) + (20 \times 0) + (30 \times 68)$$

$$= \underline{\qquad\qquad} /-$$

Step II) Degeneracy test

$5 \stackrel{?}{=} (4+3-1) = 6$

$5 \neq 6$

There is De-generacy.

There are 5 occuipied cells which can gives '6' eq^n only.

Add ε = epsilon $\varepsilon = 0$.

~~Add~~ to such a non occupied cell

1) select occuipied cell which is alone, i.e. do nt have neighbour

→ Here 15 $\lfloor 65$ hv neighbous along the rows & along the columns as 13 & 20

∴ can't select

→ RC = 30 $\lfloor 68$ is alone.

2) Now along RC (row & column) select min $Xpost^n$ cost i.e. Dummy-C cell & add ε.

3) This is trial-n-error; if 'Dummy-C' cell can't give ANS then select Next least value i.e. PC = $\lfloor 63$

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	A	B	C	
P	✓ $\boxed{60}$	$\boxed{72}$	$\boxed{63}$	$u_1 = 60$
Q	✓ $\boxed{68}$	✓ $\boxed{65}$	$\boxed{69}$	$u_2 = 68$
R	$\boxed{77}$	$\boxed{70}$	✓ $\boxed{68}$	$u_3 = 71$
DUM	$\boxed{0}$	✓ $\boxed{0}$	ϵ $\boxed{0}$	$u_4 = 3$
	$V_1 = 0$	$V_2 = -3$	$V_3 = -3$	

Step III) Optimality test

Opp cost for non-occupied cells.

PB = 72 − (60 − 3) = +15

PC = 63 − (60 − 3) = +6

RA = 77 − (71 + 0) = +6

QC = 69 − (68 − 3) = +4

RB = 70 − (71 − 3) = +2

DUM − A = 0 − (3 + 0) = −3 * start & end

∴ solⁿ nt optimum

avoid ϵ goods as corner of polygon as 13 is min at -ve sign

	A	B	C	
P	✓			
Q	13 ✓ −	28 ✓ +		
R			✓	
DUM	+ 13	7 ✓ −	✓ ε	

− 13 ⟶ 15 +   0   28

⟹

+ • ⟵ 20 −   13   7

New

	A	B	C	SUP
P	12 $\boxed{60}$	• $\boxed{72}$	• $\boxed{63}$	12
Q	• $\boxed{68}$	28 $\boxed{65}$	• $\boxed{69}$	28
R	• $\boxed{77}$	• $\boxed{70}$	30 $\boxed{68}$	30
DUM	13 $\boxed{0}$	7 $\boxed{0}$	ε $\boxed{0}$	20
DEM.	25	35	30	90
	$v_1 = 0$	$v_2 = 0$	$v_3 = 0$	

$u_1 = 60$

$u_2 = 65$

$u_3 = 68$

$u_4 = 0$

⟶

Opp. cost for non-occupied cells.

PB = +12
PC = +3
QA = +3
QC = +4
RA = +9
RB = ~~0~~ +2

$\because$ all +ve
$\therefore$ solⁿ optimum.

$\therefore$ TTC = 720 + 1820 + 2040 + 0 + 0
= 4580 ₹

Prob 14

Plants	W/H A	W/H B	W/H C	SUP
P	24	55	48	70
Q	68	42	59	80
R	72	33	49	50
DEM	60	75	85	

200

220

Total SUP = 200

Total DEM = 220

∴ As SUP exceeds the demand 1 Dummy row is with supply capacity of 20 goods is added.

	A	B	C	SUP	
P	(60) 24	x 55	(10) 48	~~70~~	24, 24, 7
Q	x 68	(75) 42	(5) 59	~~80~~	17, 17, (17)
R	x 72	x 33	(50) 49	~~50~~	16, 16, 16
DUMMY S	x 0	x 0	(20) 0	20	0, -, -
DEM	60	75	85		
	(24), (44) -	(33), 9, 9	(48), 1, 1		

Initial solⁿ by VAM

TTC = 7815 Rs.

Step II → (4+3−1) = 6 = 6 occupied cells

there is no Degeneracy.

	A	B	C	
P	24 ✓	55	48 ✓	$u_1 = 48$
Q	68	42 ✓	59 ✓	$u_2 = 59$
R	72	33	49 ✓	$u_3 = 49$
Dum	0	0	0 ✓	$u_4 = 0$
	$v_1 = 24$	$v_2 = -17$	$v_3 = 0$	

Opp cost of non-occupied cells.

PB	$55 - (-17 + 48) = \cancel{10}\ +24$	✓
QA	$68 - (59 - 24) = +33$	✓
RA	$72 - (49 - 24) = +47$	✓
RB	$33 - (49 - 17) = \cancel{33}\ +1$	
DUM-A	$0 - (\ \ - 24) = \cancel{25}\ +24$	✓
DUB-B	$0 - (-17 + 0) = +17$	

Sol^n optimum

∴ 7815 ✓

	A	B	C
P	✓		✓
Q		− ✓	✓ +
R		+	✓ −
DUM			✓

−75   5 +

+0   50 −

⇒

	A	B	C
P	✓		✓
Q		✓	✓
R			✓
DUM			✓

Prob 15

plants	P	Q	R	SUP	
A	30 27	× 35	× 45	30	8, -, -
B	× 38	60 30	× 40	60	8, 8, 8
C	20 42	× 39	50 29	70	10, 10, 3
DEM	50	60	50	160	
	11, 4, 4	5, 9, 9	11, 11, -		

TTC = 30 × 27 + 60 × 30 + 20 × 42 + 50 × 29

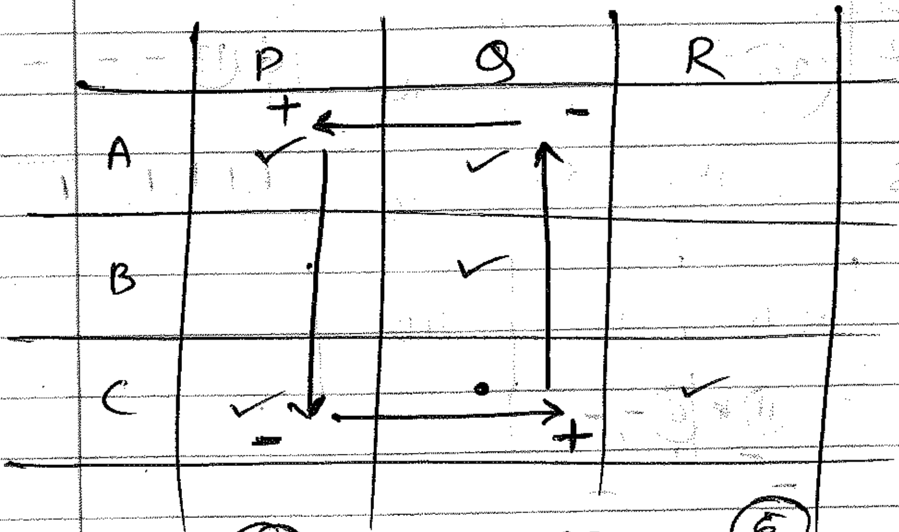
= 4900/-

Step II ⇒ (3+3-1) = 5 here 4 occupied cells

∴ there is degeneracy

	P	Q	R	
A	✓ 27	ϵ 35	45	$u_1 = 27$
B	38	✓ 30	40	$u_2 = 22$
C	✓ 42	39	✓ 29	$u_3 = 42$
	$V_1 = 0$	$V_2 = 8$	$V_3 = -13$	

AR = 45 − (27 − 13) = +
BP = 38 − (22 + 0) = +
BR = 40 − (22 − 13) = +
CQ = 39 − (42 + 8) = −ve.

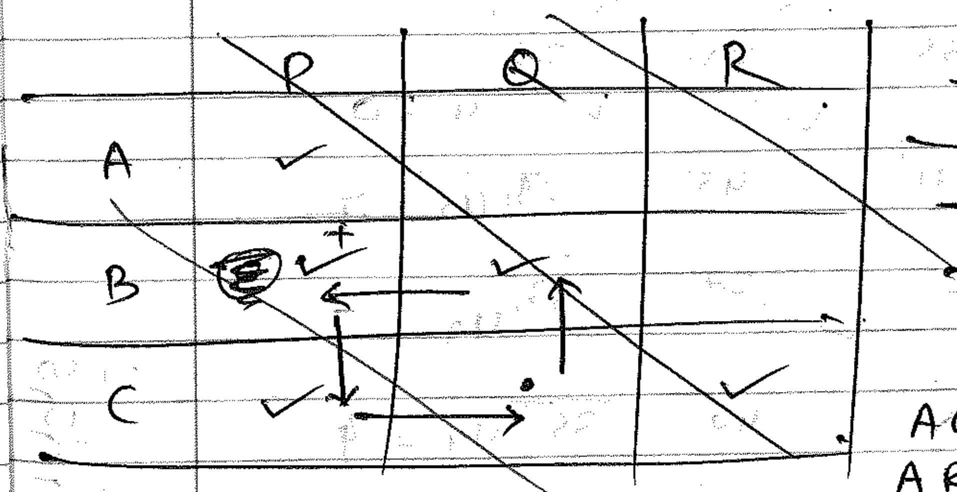


The occupied cell with ε goods gets −ve sign & hence ε goods must be shifted elsewhere

+ 30 − ε

As there −ve sign along ε ⇒ Shift ε somewhere else

− 20 , +



	P	Q	R	
A	30			27
B	ε	60		38
C	20		50	42
	0	−8	−13	

AQ = +16
AR = +31
BR = +15
CQ = +5
∴ Optimum = 4900/−

Prob 16

Dealers

Plants	P	Q	R	S	SUP	
A	15 (25)	× (35)	5 (29)	20 (23)	40	2, 4, 4, 10, –
B	× (38)	20 (31)	× (45)	× (29)	20	2, 7, 7, 7, 7
C	× (40)	× (42)	30 (27)	× (33)	30	6, 13, –, –, –
D	15 (29)	5 (28)	× (40)	× (35)	20	1, 1, 1, 1, 1
	30	25	35	20	110	
	4, 4, 4, 4, 9	3, 3, 3, 3, 3	2, 2, 11, –	6, –, –, –		

TTC = 2985

∴ Step II = $(4+4-1) = 7$ ∴ No Degeneracy

	P	Q	R	S	
A	✓ 25	35	✓ 29	✓ 23	$u_1 = 0$
B	38	✓ 31	45	29	$u_2 = 7$
C	40	42	✓ 27	33	$u_3 = -2$
D	✓ 29	✓ 28	40	35	$u_4 = 4$
	$V_1 = 25$	$V_2 = 24$	$V_3 = 29$	$V_4 = 23$	

750
145
155
115
435
810
560
2970

$AQ = 35 - (24+0) = +11$

$BP = 38 - (25+7) = +6$

$BR = 45 - (29+7) = +9$

$CP = 40 - (25-2) = +17$

$CQ = 42 - (24-2) = +20$

$CS = 33 - (23-2) = +12$

$DR = 40 - (29+4) = +7$

$DS = 35 - (23+4) = +8$

✓ $BS = 29 - (23+7) = -1$

As ~~all~~ values are +ve (-ve)

∴ sol^n is not optimum.

	P	Q	R	S	
A	30 (25) +	(35)	5 (29)	5 (23) −	$u_1 = 0$
B	(38)	5 (31) −	(45)	15 (29) +	$u_2 = 6$
C	(40)	(42)	30 (27)	(33)	$u_3 = -2$
D	(29) −	20 (28) +	(40)	(35)	$u_4 = 3$
	$V_1 = 25$	$V_2 = 24$	$V_3 = 29$	$V_4 = 23$	

-ve values = 20, 15, 20 ∴ shift 15

∴ $AQ = 35 - (24+0) = +10$

$BP = 38 - (30+6) = +7$

$BR = 45 - (29+6) = +10$

$CP = 40 - (30-2) = +17$

$CQ = 42 - (24-2) = +19$

$CS = 33 - (23-2) = +12$

$DR = 40 - (29+4) = +8$

$DS = 35 - (23+4) + 9$

Optimum TTC = $(30 \times 25) + (29 \times 5) + (31 \times 5) + (23 \times 5) + (29 \times 15) + (27+30) + (20 \times 28)$

$= 2970$

Prob 17

plants	A	B	C	D		
P	400 [1]	100 [8]	200 [4]	× [7]	700	3, 3, 4, 4
Q	× [7]	400 [2]	× [5]	× [8]	400	3, 3, 3, –
R	× [5]	200 [6]	× [9]	300 [2]	500	3, 1, 3, 3
Dem	400	700	200	300	1600	
	4, 4, –, –	4, 4, 4, 2	1, 1, 1, 5	5, –, –, –		

1) TTC = 4600/-

2) There is No Degeneracy

	A	B	C	D	
P	✓ 1	✓ 8	✓ 4	7	$u_1 = 0$
Q	7	✓ 2	5	8	$u_2 = -6$
R	5	✓ 6	9	✓ 2	$u_3 = -2$
	$V_1 = 1$	$V_2 = 8$	$V_3 = 4$	$V_4 = 4$	

PD = 7 − (4+0) = + 3
QA = 7 − (1−6) = + 12
QC = 5 − (8−6) = + 7
QD = 8 − (4−6) = + 10
RA = 5 − (1−2) = + 6
RC = 9 − (4−2) = + 7

all +ve
∴ sol^n optimum
4600/-
ANS

Prob 18

Maximizn

The company has 3 products X, Y, Z, which are sold through 4 outlets A, B, C, D.
The following table gives the profit figures when the particular product is sold thr the outlet.
Solve the Xportn problem, so as to maximize the profit.

Prodn	A	B	C	D	SUP
X	20 [17]	30 [20]	[16]	[13]	50
Y	[9]	[17]	[11]	40 [18]	40
Z	[12]	[14]	30 [15]	[13]	30
DEM	20	30	30	40	120

⇒ As the numbers represents profit, our obj is maximizn of profit. Hence the given Xportn matrix is at first converted into opp. cost matrix

select largest Profit = here 20
& All values are subtracted from 20

	A	B	C	D		Rest Same
X	20 [3]	30 [0]	• [4]	• [7]	50	3,1,1
Y	• [11]	• [3]	• [9]	40 [2]	40	1,–,–
Z	• [8]	• [6]	30 [5]	• [7]	30	1,1,3
	20	30	30	40	120	
	5,5,5	3,6 –	1,1	5 –		

TTC = NO

∴ Total Profit = ?

∴ use original profit figures.

Total Profit =

$(20 \times 12) + (30 \times 20) + (30 \times 15) + (40 \times 18)$

$= 2110/-$

Step II) No of occupied cells = 4

$(m+n-1) = 6$

There is Degeneracy.

Here two non occupied cells must be converted into occupied.

∴ ε, ε twice

	A	B	C	D	
X	3 ✓	0 ✓	4 ε ✓	7	$u_1 = 0$
Y	11	3 ε ✓	9	2 ✓	$u_2 = 3$
Z	8	6	5 ✓	7	$u_3 = 1$
	$v_1 = 3$	$v_2 = 0$	$v_3 = 4$	$v_4 = -1$	

$XD = 7 - (0-1) = +8$

$YA = 11 - (3+3) + 5$

$YC = 9 - (3+4) = +2$

$ZA = 8 - (3+1) = +4$

$ZB = 6 - (0+1) = +5$

$ZD = 7 - (-1+1) = +7$

all +ve ∴ optimum sol^n

∴ Max. profit is 2110/- Rs **ANS**

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